

Comments on "Stabilizability and Detectability of Discrete-Time Time-Varying Systems"

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Abstract—An alleged counterexample in the above paper¹ to a main result of [1] is shown not to be a counterexample in actuality.

INTRODUCTION

A recent paper¹ presented a purported counterexample to one of the main results of [1]. In [1], a definition of uniform detectability is presented for a linear finite-dimensional time-varying system, and it is then shown that bounded output-injection feedback can stabilize a system which is uniformly detectable according to the definition.

The contribution of the paper¹ is to define a linear system, assert that it is uniformly detectable in terms of the definition of [1], and then to establish that there is no bounded output-injection feedback to stabilize the system. If correct, this chain of reasoning would invalidate the claim of [1].

Below, we review the definition of uniform detectability of [1] and recall the system defined in the paper¹. We show this system is not uniformly detectable, in contrast to the assertion in the paper¹.

Accordingly, the purported counterexample is not really a counterexample to a main result of [1].

ANALYSIS OF ALLEGED COUNTEREXAMPLE

Consider the unforced linear finite-dimensional time-varying system

$$x_{k+1} = F_k x_k \quad y_k = H'_k x_k \quad (1)$$

and define the state transition matrix $\phi_{k,l}$ for $k \geq l$ by $\phi_{k,k} = I$, $\phi_{k+1,k} = F_k$ and $\phi_{k,l} = \phi_{k,k-1} \phi_{k-1,l}$. In [1], one finds:

Definition: The pair $[F_k, H_k]$ is uniformly detectable if there exist integers $s, t \geq 0$ and constants d, b with $0 \leq d < 1, 0 < b < \infty$ such as whenever

$$\|\phi_{k+t,k} \xi\| \geq d \|\xi\| \quad (2)$$

for some ξ and k , then

$$\xi' M_{k+s,k} \xi \geq b \xi' \xi \quad (3)$$

where

$$M_{k+s,k} = \sum_{i=k}^{k+s} \phi'_{i,k} H_i H'_i \phi_{i,k} \quad (4)$$

Now consider the following system

$$F_{2j} = \begin{pmatrix} 0 & 0 \\ 4 & 0 \end{pmatrix} \quad F_{2j+1} = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \quad (5)$$

$$H'_{2j} = [0 \quad 1] \quad H'_{2j+1} = [0 \quad 0] \quad (6)$$

In the paper¹, it is asserted that the requirements of the definition are fulfilled with $s = 0, t = 1, d = 3/4$, and $b = 3/64$.

Choose $k = 2j, \xi = [1 \ 0]'$. Then with $t = 1$

$$\left\| \phi_{k+1,k} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\| \geq \frac{3}{4} \left\| \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\|. \quad (7)$$

Also, with $k = 2j$ and $s = 0$

$$M_{k+s,k} = H_{2j} H'_{2j} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

and then with $\xi = [1 \ 0]'$, evidently $\xi' M_{k+s,k} \xi = 0$, and so there cannot hold

$$\xi' M_{k+s,k} \xi \geq b \xi' \xi \quad (8)$$

with $b = 3/64$. Yet uniform detectability requires, since (7) holds, that (8) must hold.

Hence, the system defined by (5) and (6) is not uniformly detectable for the reason asserted in the paper¹, i.e., by virtue of the satisfaction of the definition for $s = 0, t = 1, d = 3/4$, and $b = 3/64$. This still leaves open the question of whether other choices of s, t, d , and b could render it uniformly detectable.

Simple calculations reveal that

$$M_{2j+s,2j} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \forall s \geq 0.$$

Now for $k = 2j$, we have

$$\phi_{k+1,k} = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$\phi_{k+2,k} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\phi_{k+3,k} = \begin{bmatrix} 0 & 0 \\ 8 & 0 \end{bmatrix}$$

$$\phi_{k+4,k} = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}.$$

Suppose, to obtain a contradiction, that particular values $\bar{t}, \bar{s}, \bar{d} < 1$, and $\bar{b} > 0$ will work in the uniform detectability definition. Choose $\xi = [1 \ 0]'$. Evidently, no matter what \bar{t} is, there holds with k even

$$\|\phi_{k+\bar{t},k} \xi\| > \|\xi\| > \bar{d} \|\xi\|.$$

But also $\xi' M_{k+\bar{s},k} \xi = 0$ for any \bar{s} , which means that $\xi' M_{k+\bar{s},k} \xi \geq \bar{b} \xi' \xi$ fails. Hence it is impossible, through variation of the defining constraints s, t, d , and b away from those chosen in the paper¹ to secure uniform detectability.

REFERENCES

- [1] B. D. O. Anderson and J. B. Moore, "Detectability and stabilizability of time-varying discrete-time linear systems," *SIAM J. Contr. Optimiz.*, vol. 19, no. 1, pp. 20-32, 1981.

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¹J. C. Engwerder, "Stabilizability and detectability of discrete-time time-varying systems," *IEEE Trans. Automat. Contr.*, vol. AC-35, no. 4, pp. 425-429, Apr. 1990.