

Arguments for Probabilism - or Non-Probabilism?

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- My schizophrenia regarding probabilism.

The two theses of probabilism

1. Beliefs come in degrees - call them *degrees of belief*.
2. An agent's degrees of belief should conform to the laws of probability, on pain of irrationality.

What are “the laws of probability”?

- “They’re simply Kolmogorov’s axioms”.
 - That’s too quick. “The laws of probability” could mean more, it could mean less, and it could mean something rather different.

Kolmogorov's axioms

1. $P(X) \geq 0$. (Non-negativity)
2. $P(\Omega) = 1$. (Normalization)
3. $P(A \cup B) = P(A) + P(B)$, if $A \cap B = \emptyset$. (Additivity)

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- 3' $P(\cup A_n) = \sum P(A_n)$ if the A_i are (pairwise) disjoint
(Countable additivity)

The numerical axioms, questioned

1. $P(X) \geq 0$. (Non-negativity)
 - Negative probabilities?

2. $P(\Omega) = 1$ (Normalization)
 - Unbounded probabilities?

3. $P(A \cup B) = P(A) + P(B)$, if $A \cap B = \emptyset$. (Additivity)
 - Non-additive probabilities?

- 3' $P(\cup A_n) = \sum P(A_n)$ if the A_i are (pairwise) disjoint
(Countable additivity)
 - De Finetti?

Conditional probability

$$P(X|Y) = P(X \cap Y)/P(Y), \text{ provided } P(Y) > 0.$$

- Primitive conditional probabilities?

- The Big Four arguments in the philosophical literature for ‘probabilism’ are unconcerned with these matters:
 - Dutch Book argument
 - Representation theorem
 - Calibration
 - Gradational accuracy
- At most, they seek to justify *conformity to Kolmogorov’s axioms of non-negativity, normalization, and finite additivity* as a rational requirement.
- Still, even that goal is not as easily achieved as one might think.

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If your credences violate the laws of probability, then **there exists** something '**bad**'.

The 'badness' is supposed to display some kind of irrationality in you.

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 - Each argument is invalid, unless we are careful to state the theorem just right.

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 - The 'badness' is supposed to display some kind of irrationality in you.
 - That's not obvious to me.
 - Each argument concludes with probabilism.
 - Each argument is invalid, as traditionally stated in the philosophical literature.
 - In each case there is a 'mirror-image' theorem that threatens to undercut probabilism.

The original argument provides good news for probabilism, but the mirror-image theorem provides bad news. Our final verdict can only be given after weighing the good and the bad news. If that verdict does not side with probabilism, then the probabilist had better find new sources of good news.

1. The Dutch Book Argument

Begin with the betting interpretation of credences.

A *Dutch Book* is a set of bets, each of which you consider fair, which collectively guarantee that you will lose.

Example

Suppose you assign:

$$P(\text{Rain}) = 0.3$$

$$P(\text{Not rain}) = 0.4$$

I buy from you a bet that pays a dollar if rain for \$0.3.

I buy from you a bet that pays a dollar if not rain for \$0.4.

I have paid \$0.7. Whatever happens, I win \$1. You have lost \$0.3 for sure.

1. The Dutch Book Argument

Dutch Book Theorem

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Dutch Book argument.

Conclusion:

Rationality requires you to conform to the laws of probability.

(Note the logical form of the theorem: a conditional with an existentially quantified consequent.)

The argument is *clearly* invalid as it stands...

We need to add the Converse Dutch Book theorem:

If you obey probability theory, then there does not exist a Dutch Book against you.

So far, so good for probabilism.

But there is a mirror-image theorem, and argument

...

A *Czech Book* is a set of bets, each of which you consider fair, which collectively guarantee that you will WIN.

Czech Book Theorem

If you violate probability theory, then there exists a set of bets, each of which you consider fair, and which collectively guarantee your GAIN.

Czech Book Argument.

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If you violate probability theory, then there exists a set of bets, each of which you consider fair, and which collectively guarantee your GAIN.

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Conclusion:

Rationality requires you to **violate** the laws of probability.

The argument is *clearly* invalid as it stands...

We need to add the Converse Czech Book theorem:

If you obey probability theory, then there does *not* exist a Czech Book for you.

(Proof is trivial.)

So far, so bad for probabilism...

2. Representation theorem argument

The centerpiece of the argument for probabilism from representation theorems is some version of the following theorem, which I will not dispute:

Representation theorem

If your preferences satisfy certain conditions, then there exists a representation of you as an expected utility maximizer, relative to some probability and utility function.

(Or contrapositive)

We need to bridge the gap from your merely being *representable* as obeying the probability calculus, to your *actually obeying* it (assuming that you are suitably rational).

The argument for probabilism (e.g. Maher)

1. (Interpretivism) You have a particular probability and utility function iff attributing them to you provides an interpretation that is:
 - (i) sufficiently good at making sense of you and
 - (ii) better than any competing interpretation.

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Thus, (generalizing what has been established about ‘you’ to ‘all rational persons’)

- Conclusion: “[All] rational persons have probability and utility functions”.

Objections

I question every premise except 3 (the theorem), and above all the inference from the premises to the conclusion.

The invalid inference to probabilism

- To validly infer probabilism, we need also to show that *no other interpretation is as good* as the expected utility representation.
- For all that has been said, there may be *other* perfect interpretations out there (whatever that means).

The invalid inference to probabilism

- The following ‘mirror-image’ theorem is equally true:

If you satisfy the same rationality constraints on preferences, then you can be interpreted as maximizing non-expected utility, some rival to expected utility.

In particular, you can be interpreted as having degrees of belief that *violate* probability theory, but you compensate for this with some other rule for combining your degrees of belief with utilities.

Zynda (2000) shows that you *can* be represented with a sub-additive belief function, and a corresponding combination rule.

For all that the representation theorem argument shows, this rival interpretation may also be “perfect”. So we do not have a valid inference to probabilism.

3. The calibration argument

Motivation: it is good when your credences agree with corresponding relative frequencies.

Example: coin tossing.

Calibration of a weather forecaster

- To be perfectly calibrated, the proportion of rainy days, among days on which you announce a probability of 0.8 must be 0.8; and likewise for all other probabilities that you announce over the course of the year.
- More generally, we can ‘score’ you according to a calibration index.

Calibration theorem

If c violates the laws of probability then there is a probability function that is better calibrated than c under every logically consistent assignment of truth-values to propositions.

The argument for probabilism is based on the calibration theorem. If you are incoherent, then you could be better calibrated come what may by being coherent instead.

Perfect calibration, moreover, is supposed to be A Good Thing.

Thus, the argument concludes, you should be coherent.

The calibration argument is invalid

The following ‘mirror-image’ theorem is also true:

If c violates the laws of probability then there is a NON-probability function that is better calibrated than c under every logically consistent assignment of truth-values to propositions.

- If you are incoherent, then you know *a priori* that you could be better calibrated by *staying incoherent*, but in some other way.
- To be sure, the mirror-image theorem gives you no advice as to which non-probability function you should move to. But nor did the calibration theorem give you advice as to which probability function you should move to.
- Moreover, for all we have heard so far, you might *worsen* your calibration index, come what may, by moving from a non-probability function to a ‘wrong’ probability function.
- City analogy

- Given that you can improve your situation *either* by moving to some probability function *or* by moving to some other non-probability function, why should you move to a probability function?
- An answer: Non-probability functions are *unstable* stopping points.
- But then we need to show that probability functions are *stable* stopping points.

We need:

If c OBEYS the laws of probability then there is NOT another function that is better calibrated than c *under ever logically consistent assignment of truth-values to propositions.*

i.e. probability functions are exactly the functions that are not calibration-dominated by any other function.

Nothing less will save the calibration argument.

4. The Gradational Accuracy Argument

Motivation: traditional epistemology is concerned
with *truth*...

Example: coin tossing.

4. The Gradational Accuracy Argument

Gradational accuracy theorem

“if c violates the laws of probability then there is a probability function c^+ that is strictly more accurate than c *under every logically consistent assignment of truth-values to propositions.*” (Joyce 2004, 143)

Joyce gives the following account of the argument. It:
relates probabilistic consistency to the *accuracy* of graded beliefs. The strategy here involves laying down a set of axiomatic constraints that any reasonable gauge of accuracy for confidence measures should satisfy, and then showing that probabilistically inconsistent measures are always less accurate than they need to be. (142)

- Let us agree that the axioms on inaccuracy measures are acceptable. But I don't agree that the demand for probabilistic consistency follows.

- We have a ‘mirror-image’ theorem:

If c violates the laws of probability then there is a NON-probability function c^+ that is strictly more accurate than c *under every logically consistent assignment of truth-values to propositions.*

(The trick here is that c^+ is incoherent, but a little *less* incoherent than c .)

- Why, then, are you under any rational obligation to move instead to a coherent function?
- Answer (as before): Stopping at a non-probability function will give you no rest.

But then we need to shore up the argument with a further theorem: probability functions DO provide stable stopping points.

If c OBEYS the laws of probability then there is NOT another function c^+ that is strictly more accurate than c under every logically consistent assignment of truth-values to propositions.

In other words, *probability functions are exactly the functions that are not accuracy-dominated by some other function.*

But even that may not be enough. It is not much use to the incoherent agent to know that *some* probability functions would be better than her current state of opinion, if she doesn't know *which* probability functions they are.

Recall the city analogy.

- At this point it might be tempting to say: moving to *any* probability function would improve her situation. For *wherever* she moves, she will no longer have to worry about being accuracy-dominated by some other function, come what may.
- But this just shows that the ‘come what may’ criterion is less significant than one might think.
- Consider an agent who is just slightly incoherent, but almost perfectly in step with the truth values of the actual world. ... Compare her to the coherent agent who is completely out of step...

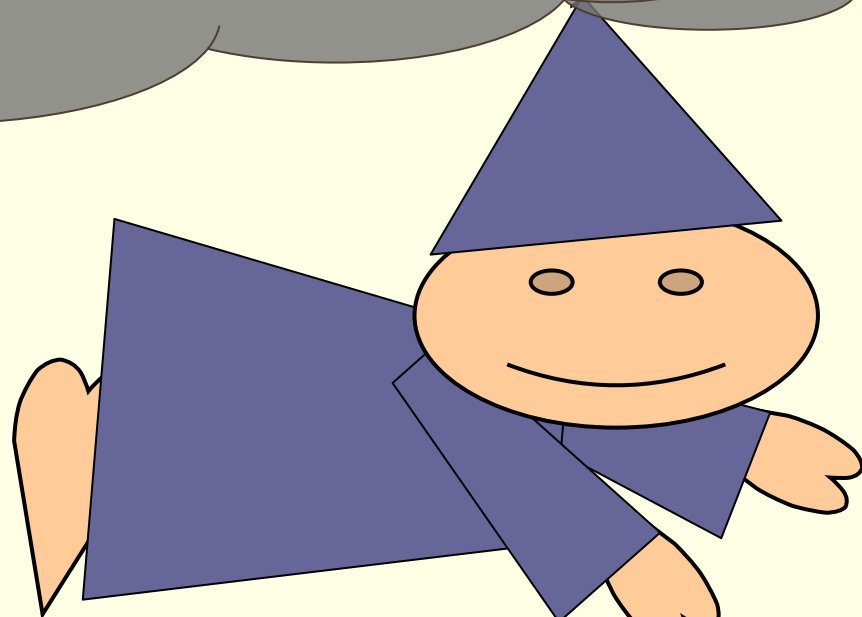
- Who would you rather be?
- This is particularly telling when the goal is *truth*, or its probabilistic analogue. Compare traditional epistemology, and its concern with beliefs that are *actually true* (never mind their status in other possible worlds). Epistemologists typically care not about ‘come what may’, but rather ‘come what *is*’.

- The fact that in some *other* possible world it is snowing around the equator does not mean that you should pack your skis when you travel to Singapore.
- Likewise, don't start believing that you are a brain in a vat, or living in the matrix!

Conclusion

- Probabilism provides us with many fruits.
 - It gives an integrated epistemology and decision theory.
 - See Howson & Urbach, Earman...
- That may be the best argument for it.
- Concluding schizophrenia.

THE END
(thanks for your attention)



Saving the Dutch Book argument

- For some reason, *many* of the presenters, both sympathetic and unsympathetic, of the Dutch Book argument focus solely on bets bought or sold at exactly your fair prices, bets that you consider fair.
- The list, containing many a luminary in the philosophy of probability, includes: Adams (1962), Adams and Rosenkrantz (1980), Armendt (1992), Baillie (1973), Carnap (1950, 1955), Christensen (1991, 1996, 2001), de Finetti (1980), Döring (2000), Earman (1992), Gillies (2000), Howson and Urbach (1993), Jackson and Pargetter (1976), Jeffrey (1983, 1992), Kaplan (1996), Kemeny (1955), Kennedy and Chihara (1979), Lange (1999), Lehman (1955), Maher (1993), Mellor (1971), Milne (1990), Rosenkrantz (1981), Seidenfeld and Schervish (1983), Skyrms (1986), van Fraassen (1989), Weatherson (1999), Waidacher (1997), and Williamson (1999). And my former self.

- But bets that you consider fair are not the only ones that you accept; you also accept bets that you consider *favourable*—that is, better than fair. You are prepared to sell a given bet at higher prices, and to buy it at lower prices, than your fair price.

Let's rewrite the theorems, replacing 'fair' with 'fair-or-favourable' throughout, and see what happens:

Dutch Book theorem, revised:

If you violate the probability calculus, there exists a set of bets, each of which you consider fair-or-favourable, that collectively guarantee your loss. TRUE

Converse Dutch Book theorem, revised:

If you obey the probability calculus, there does not exist a set of bets, each of which you consider fair-or-favourable, that collectively guarantee your loss. TRUE

Czech Book theorem, revised:

If you violate the probability calculus, there exists a set of bets, each of which you consider fair-or-favourable, that collectively guarantee your gain. TRUE.

Converse Czech Book theorem, revised:

If you obey the probability calculus, there does not exist a set of bets, each of which you consider fair-or-favourable, that collectively guarantee your gain. FALSE!

...

Ramsey got it right!

“Having degrees of belief obeying the laws of probability implies a further measure of consistency, namely such a consistency between the odds acceptable on different propositions as shall prevent a book being made against you” (1980, 42).

Note that Ramsey does not say that all of the bets in the book are individually considered fair by the agent. He leaves open the possibility that some or all of them are considered better than fair; indeed “acceptable” odds sounds synonymous with “fair-or-favourable” odds. After all, one would accept bets not only at one’s fair odds, but also at better odds.

Ramsey again:

[The agent] will take a bet at any better odds than those corresponding to his state of belief; in fact his state of belief is measured by the odds he will just take;... (1980, 37).

- Many subsequent authors restricted the Dutch Book argument solely to fair odds. They did Ramsey, and the Dutch Book argument itself, a disservice.

- This is not merely a tiny, cosmetic point.
 - Tribute to Ramsey.
 - The argument stated in terms of ‘fair’ bets was invalid - period!
 - It shows how careful one has to be in stating the central theorem.
 - We will see this again and again with the other arguments for probabilism.

- A nice book is a set of bets, each of
- which looks **at least** fair to the agent individually, and which
- together guarantee a gain.

- It is **not** true that for any incoherent agent INC, there's a
- coherent agent CO such that CO enjoys all of the nice books that INC
- enjoys. Here is an example to show this.

- Notation: "[n if P]" is a ticket worth \$n if P is true, and worth nothing otherwise.

On Higher-Order and Free-Floating Chances

Justin C. Fisher

University of Arizona – Dept of Philosophy

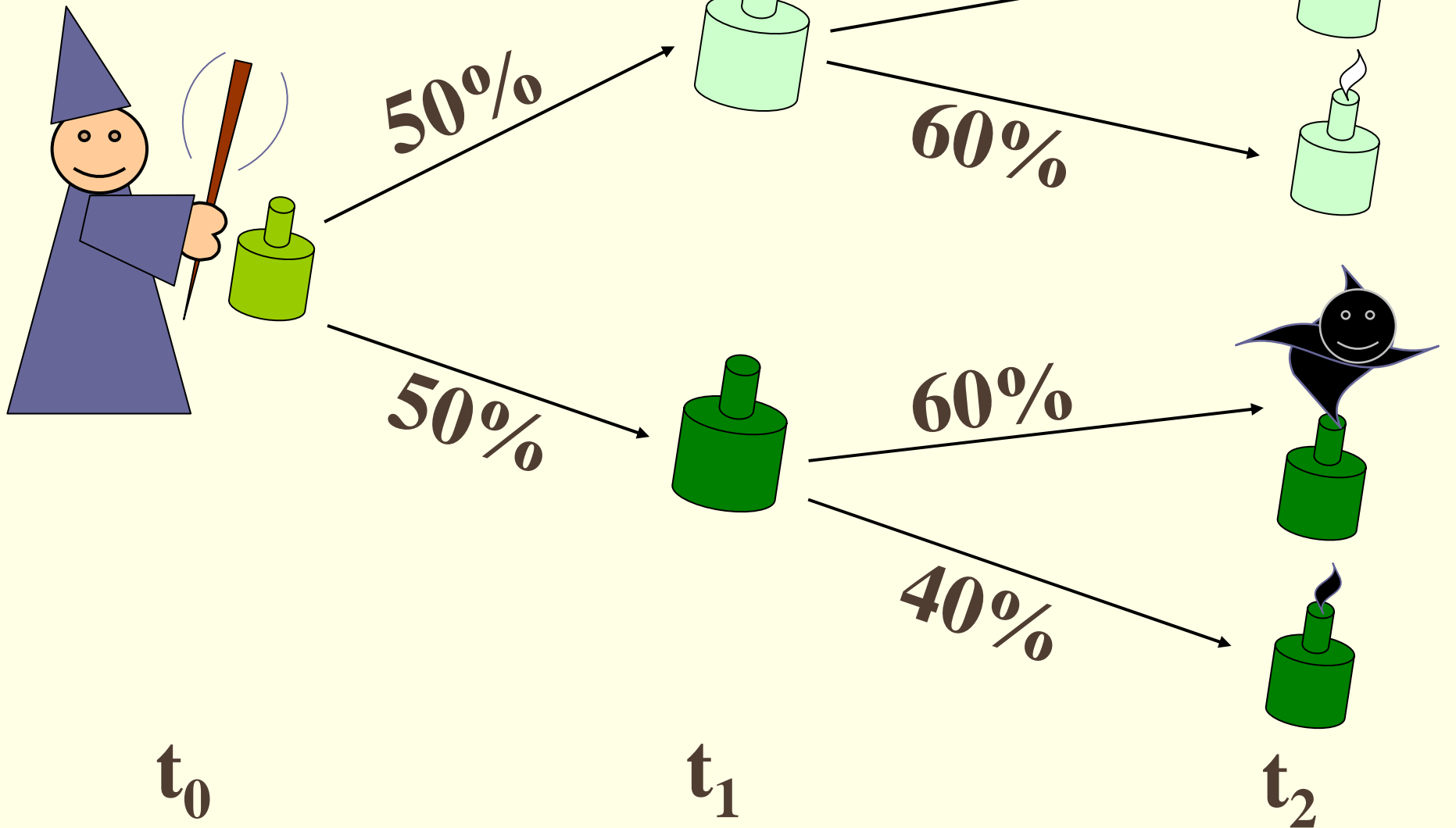
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<http://www.u.arizona.edu/~jcfisher>

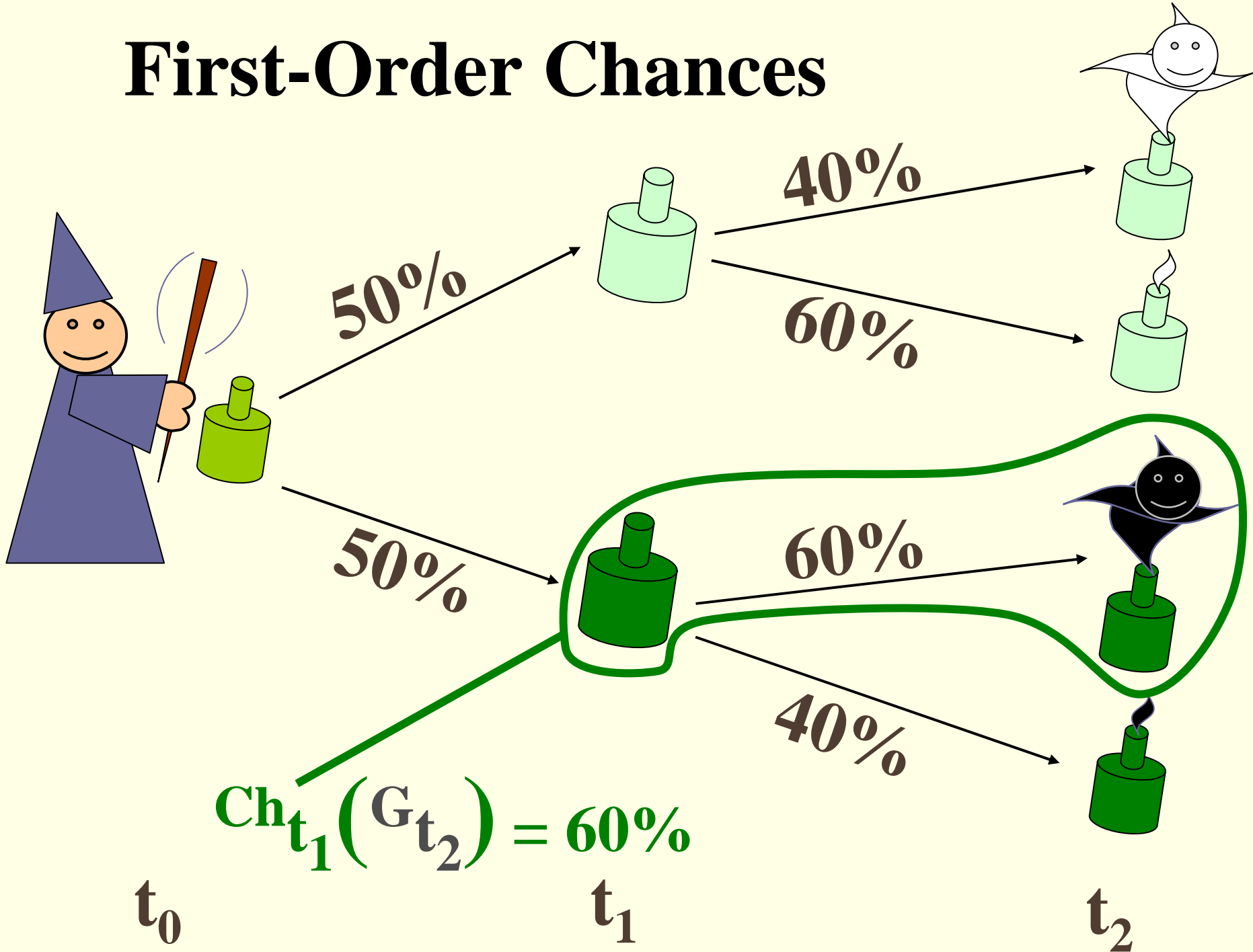


**1. First- and Higher-Order
Chances.**

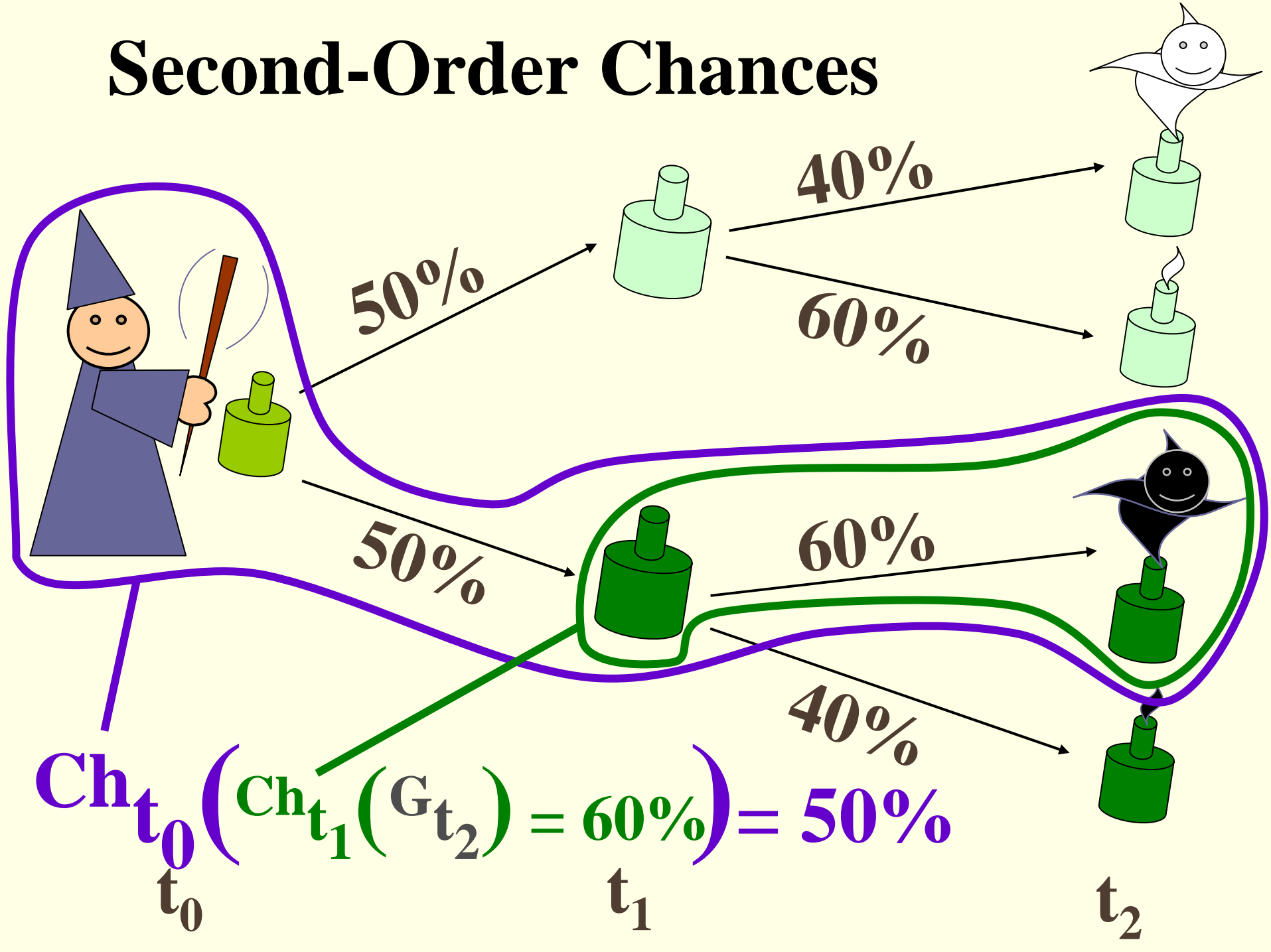
Merlin's Enchantment



First-Order Chances



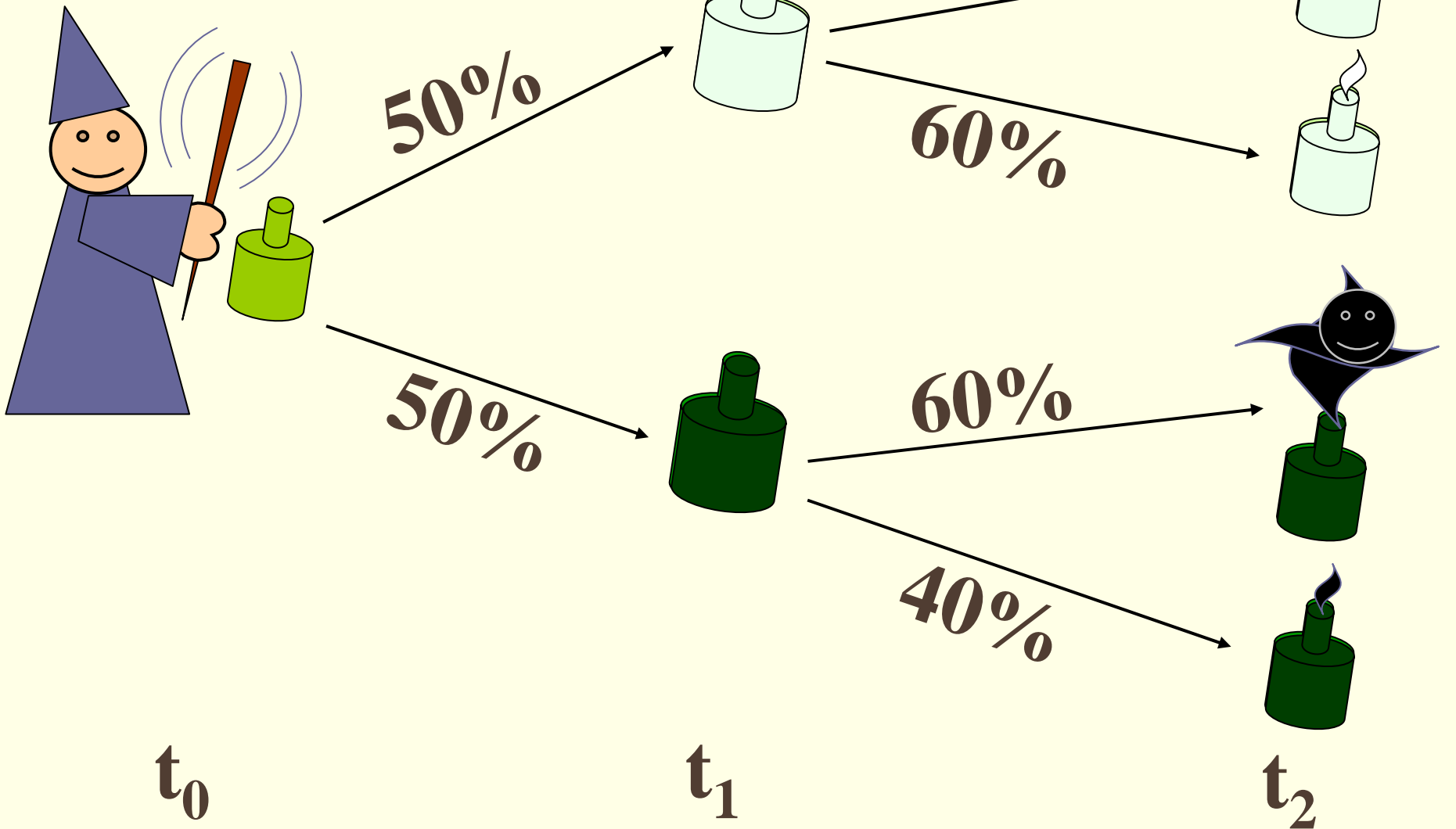
Second-Order Chances



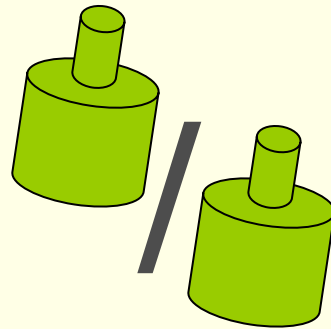
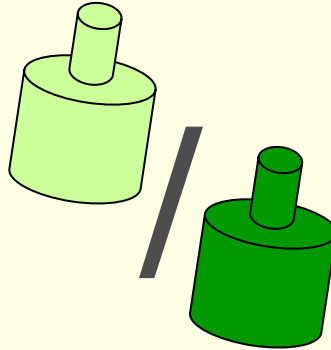
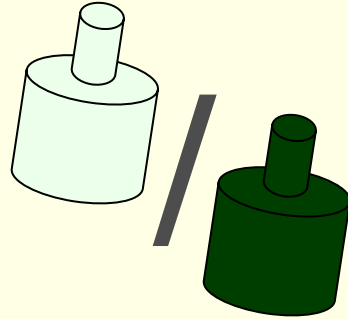


2. Free-Floating Chances.

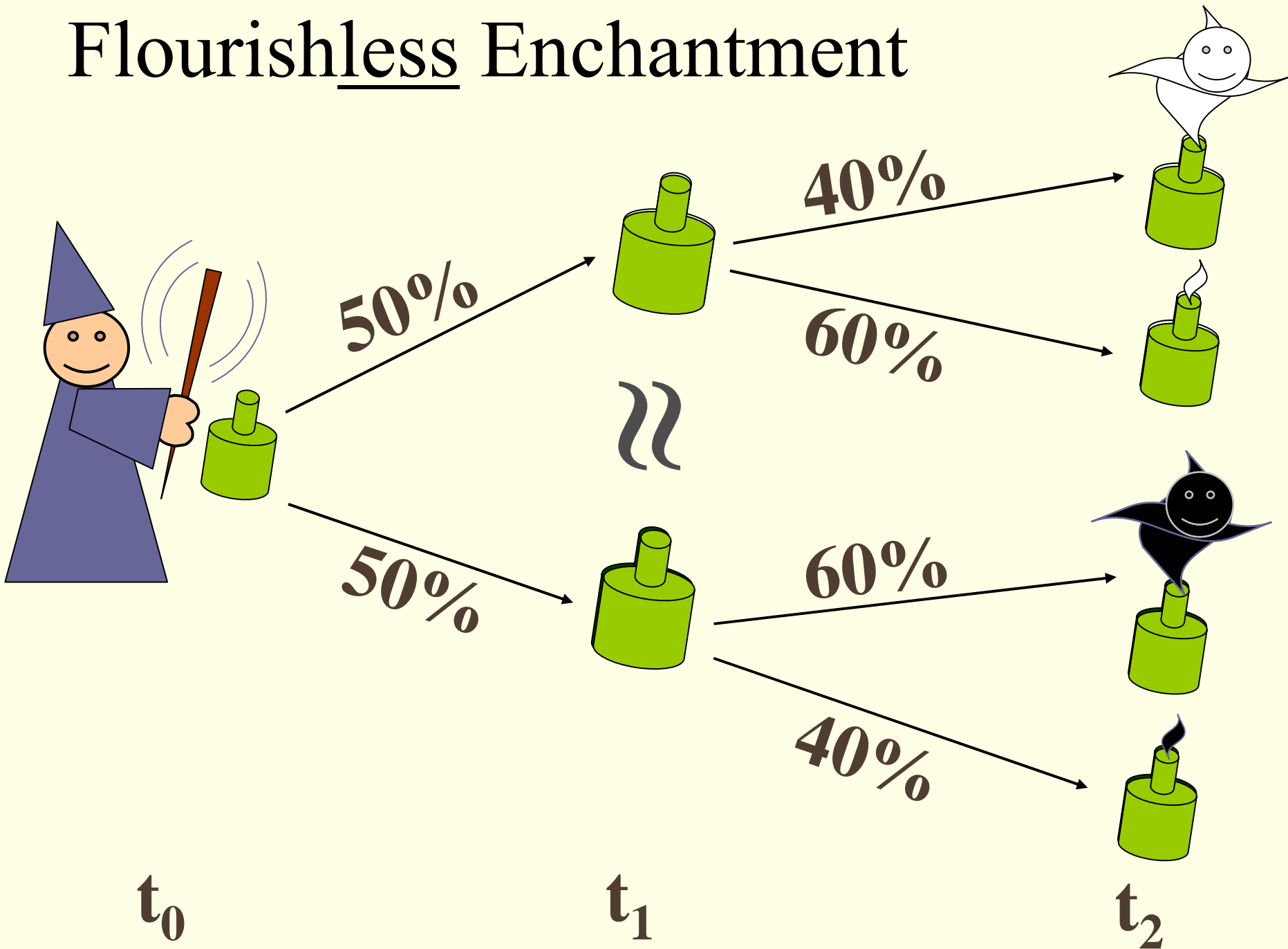
Flourishful Enchantment



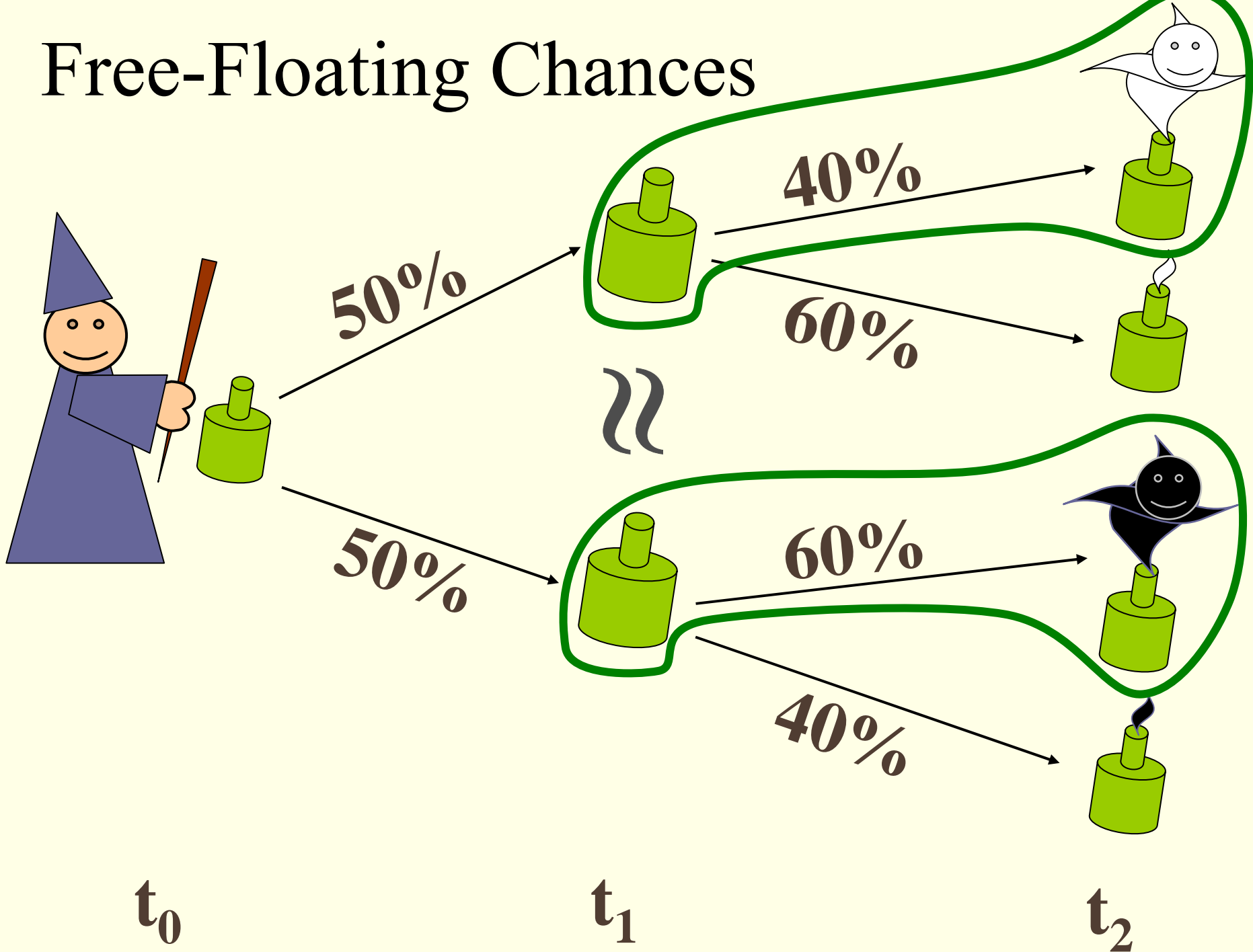
Effects of Flourish



Flourishless Enchantment



Free-Floating Chances





**3. Support for the Intuitive
Assessment.**

Lewis' Best System Account

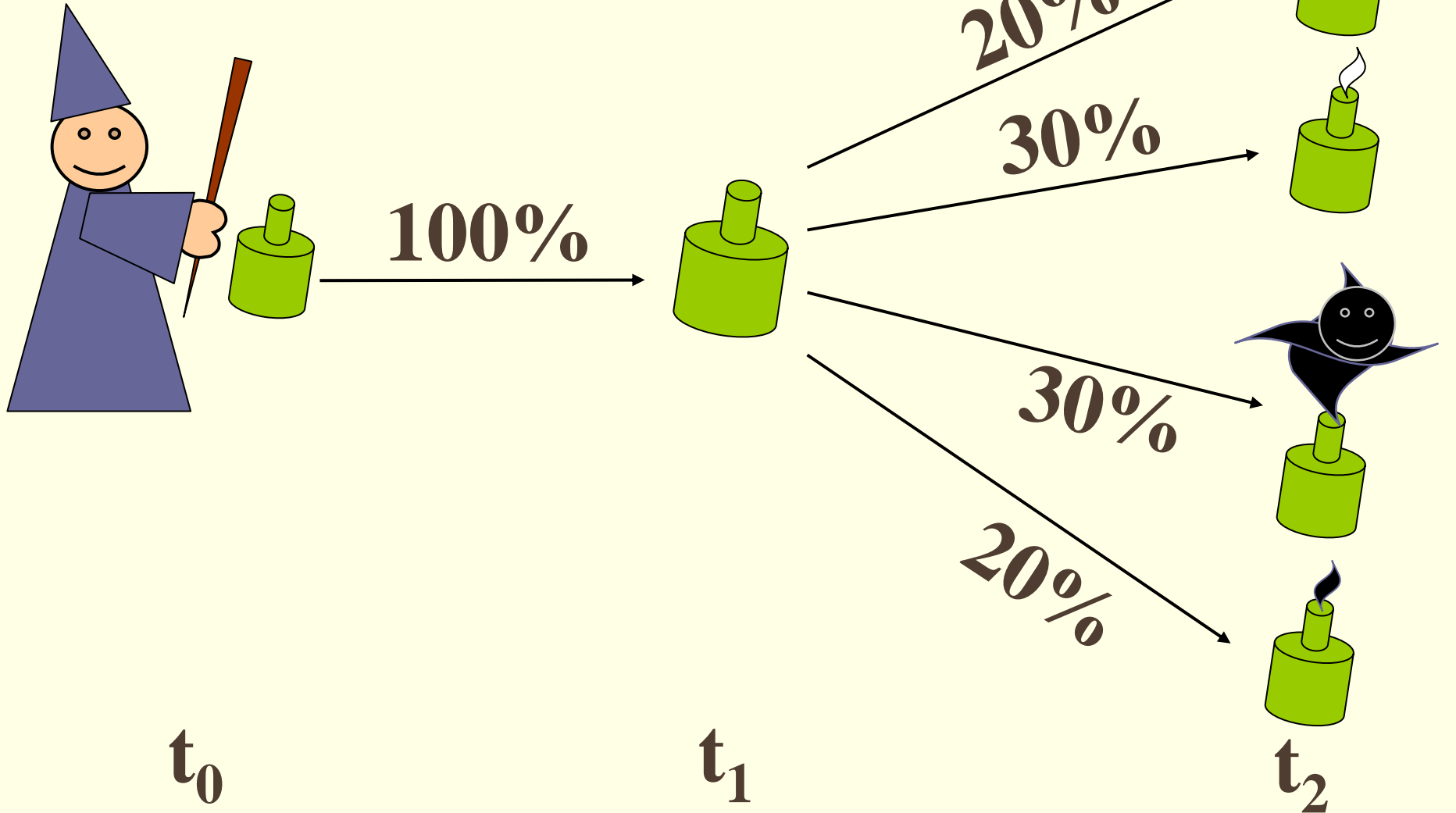
The chances that obtain in the world are those posited in the systematic description that provides the best balance between:

Strength – saying lots of truths about the distribution of categorical (non-chancy) properties at various times.

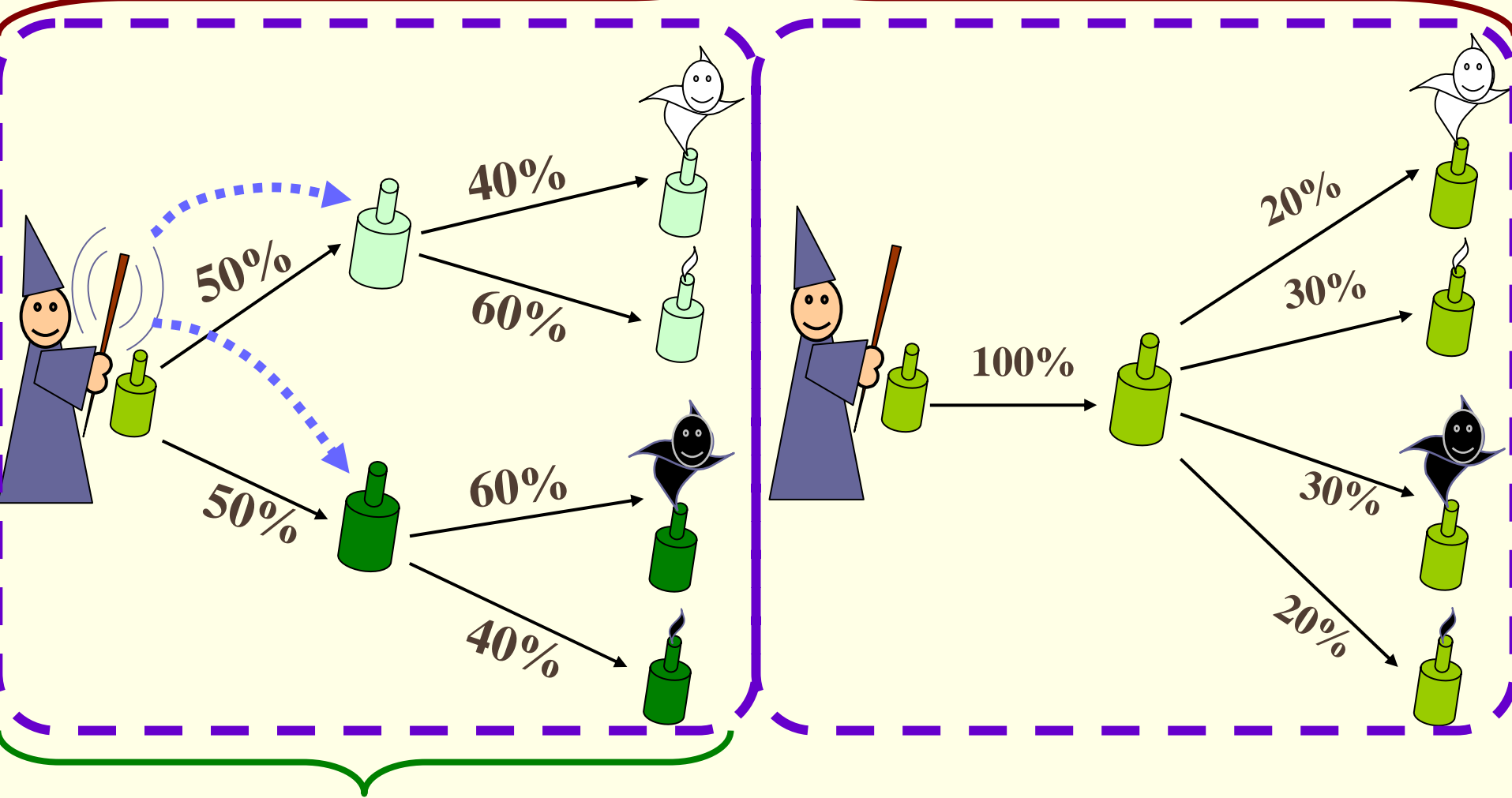
Fit – attributing chances that fit actual frequencies

Simplicity

What if we posit no Free-Floating Chances?



Positing no Free-Floating Chances



Positing Free-Floating Chances

Two Readings of Best System Accounts

Metaphysical reading – for certain chances to obtain *just is* for a certain systematic description to be best.

Epistemic reading – chances are irreducible, but strength, fit, and parsimony lend credibility to hypotheses about them.

Both readings weigh in favor of the possibility of free-floating chances.



4. Three Principles Violated.

Lawful Magnitude Principle (LMP)

All chances at a given time are determined by the laws of nature together with the complete history of the world's categorical (i.e., non-chancy) properties up to and including that time.

(see Schaffer 2003)

Chances evolve by CONDitionalization

“A later chance distribution comes from an earlier one by conditionalizing on the complete history [H] of the interval in between” (Lewis 1986, pg. 201).

$$\text{Ch}_{t_1}(G_{t_2}) = \text{Ch}_{t_0}(G_{t_2} \mid H)$$

The Principal Principle (PP)

If there is a chance at t_1 of future event G_{t_2} , this chance must be equal to the rational credence (or degree of belief) that a *suitably well-informed agent* would give to G_{t_2} .

(see Lewis 1986, 1994)

Two Formulations of PP

Original: a SWI-Agent knows all the chances at t_1 and has no inadmissible info about the future.

“Reformulated”: an SWI-agent knows only the laws of nature and the full categorical history leading up to and including t_1 .

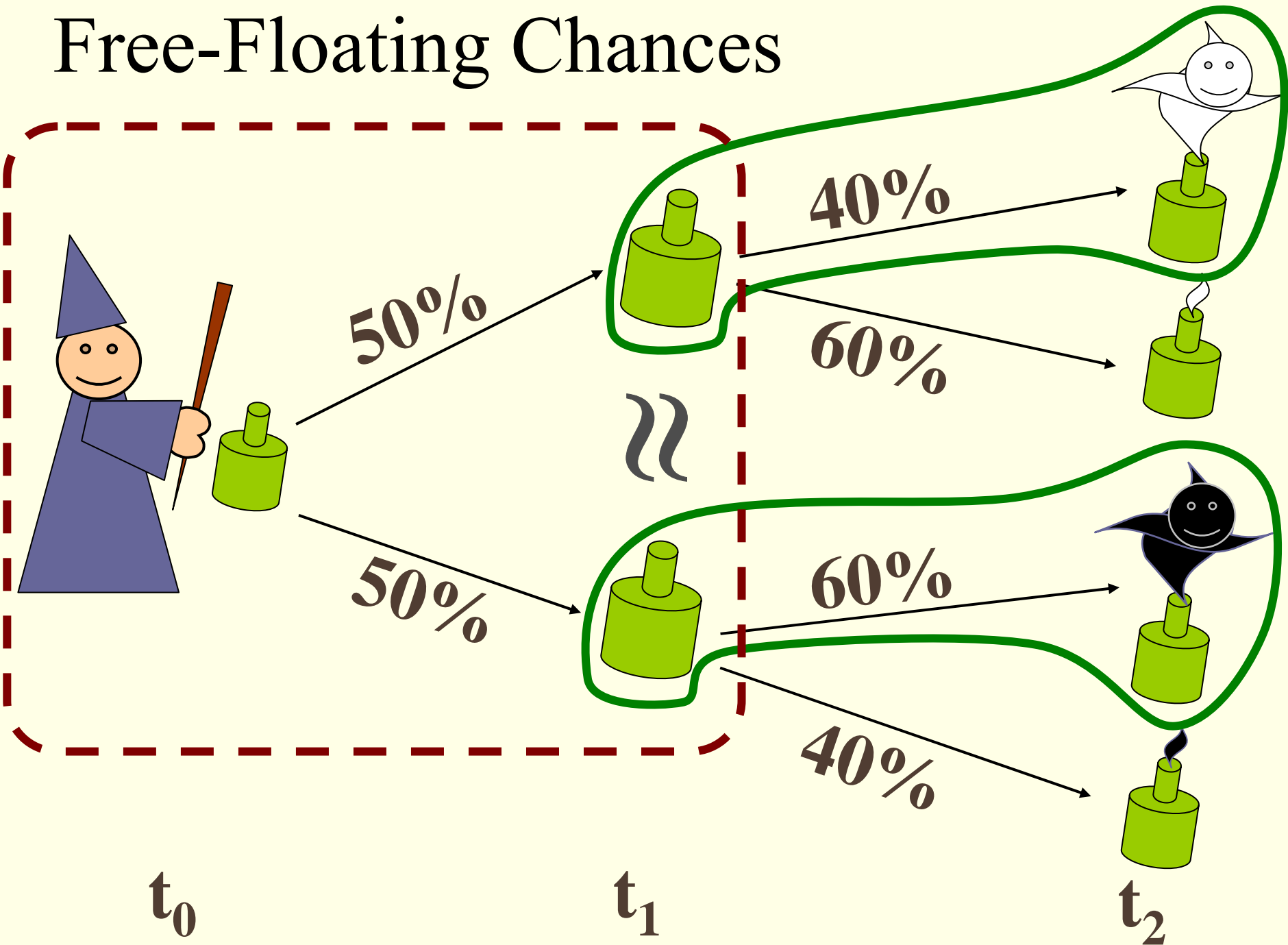
LMP, COND, and PP are alike

Each demands a tight tethering between chances and the sequence of categorical properties leading up to them.

Given a full set of prior chances, a full set of natural laws, and a categorical history leading up to and including t_1 , the chances at t_1 must be uniquely determined.

But not so in Merlin's case!

Free-Floating Chances





5. What to Do?

Start with LMP

Sure it might be *nice* if the world displayed the neat dynamical relationship LMP describes between laws, first-order chances, and categorical properties.

We might even be lucky enough to live in a nice world like this.

Still, it is *at least coherent* to suppose that in some possible worlds – worlds like Merlin's – the relationship between laws, chances, and categorical properties isn't quite so straightforward.

LMP as a Default Hypothesis

LMP is relatively simple and easy to work with.

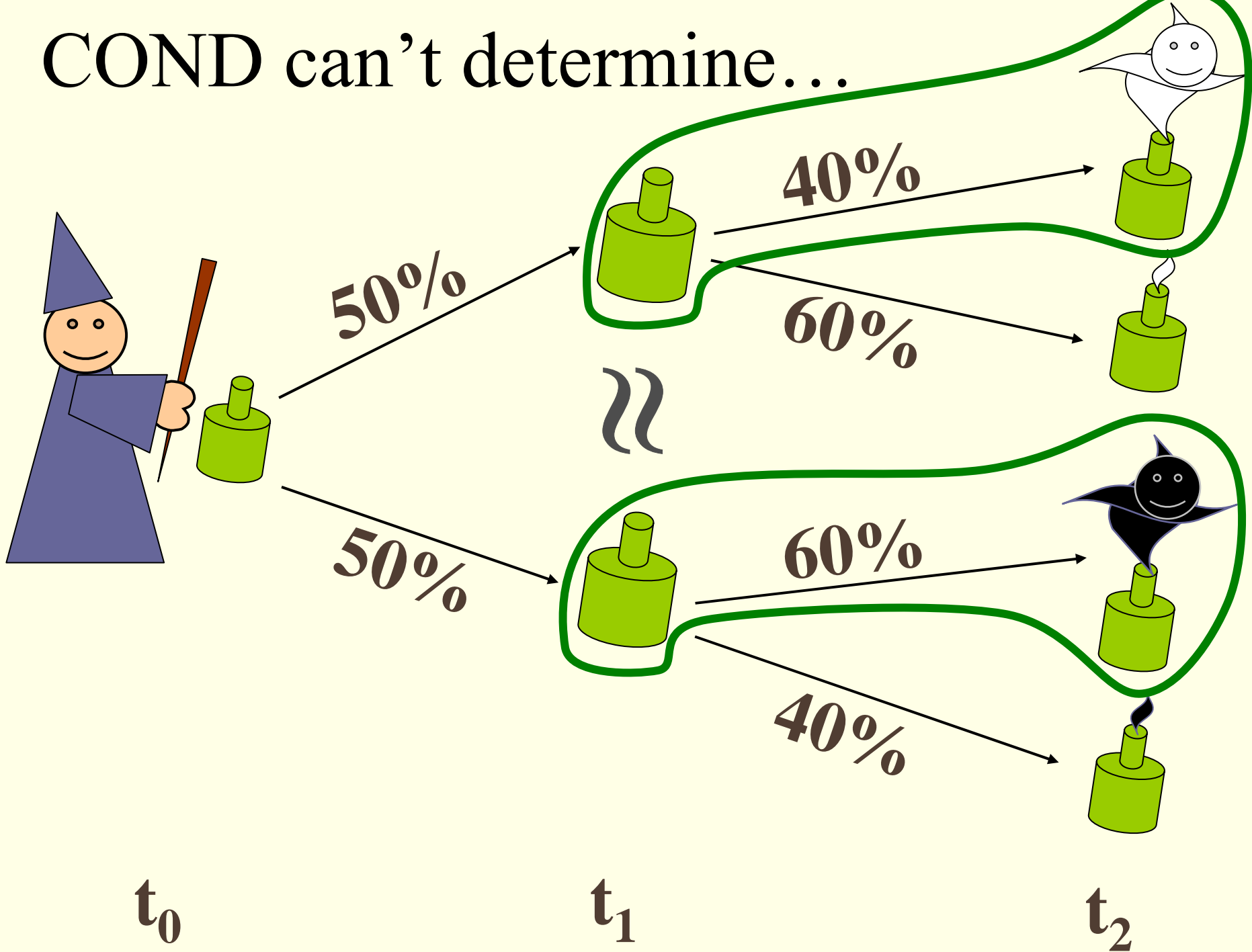
Until empirical findings press us to reject it, we should think it probably holds *in our world*.

But there are possible worlds (like Merlin's) in which it fails, and we shouldn't ignore these in thinking about the metaphysics of chances.



**6. COND as a
Default Hypothesis**

COND can't determine...



Lange's suggestion

Chances are somewhat sparse. Chances sometimes 'pop into existence', but once there, they evolve by COND.

At t_0 there is no chance of a genie coming out of the bottle at t_2 – for if there were it couldn't evolve by COND.

Instead the chance of a genie coming out at t_2 (either 40% or 60%) pops into existence at t_1 .

Problems for Lange

‘Popping into existence’ is weird.

Proposed degree of sparseness is not independently motivatable, and is apparently ad hoc.

Common usage allows that we may know that the chance of a Merlin-style enchantment yielding a genie is 50% without worrying about whether or not there are categorical indicators of any changes in this chance along the way.

COND as a Default Hypothesis

In the simplest chancy worlds, first-order chances evolve in accordance with COND.

But in more complicated worlds (like Merlin's) the laws governing the evolution of first-order chances will be more complicated:

- Sometimes first-order chances will evolve in lock-step with categorical properties (as per COND),
- But sometimes they will (temporarily) be jostled free of categorical properties under the stochastic guidance of second-order chances.

A Hierarchy of COND-Hypotheses

For all I've said about Merlin's world, it would be reasonable to guess that at least the *second-order* chances there evolve in accordance with COND.

But we could imagine even more complicated worlds where second-order chances might also sometimes be jostled by even-higher-order chancy events.



**7. A More Principled
Principal Principle**

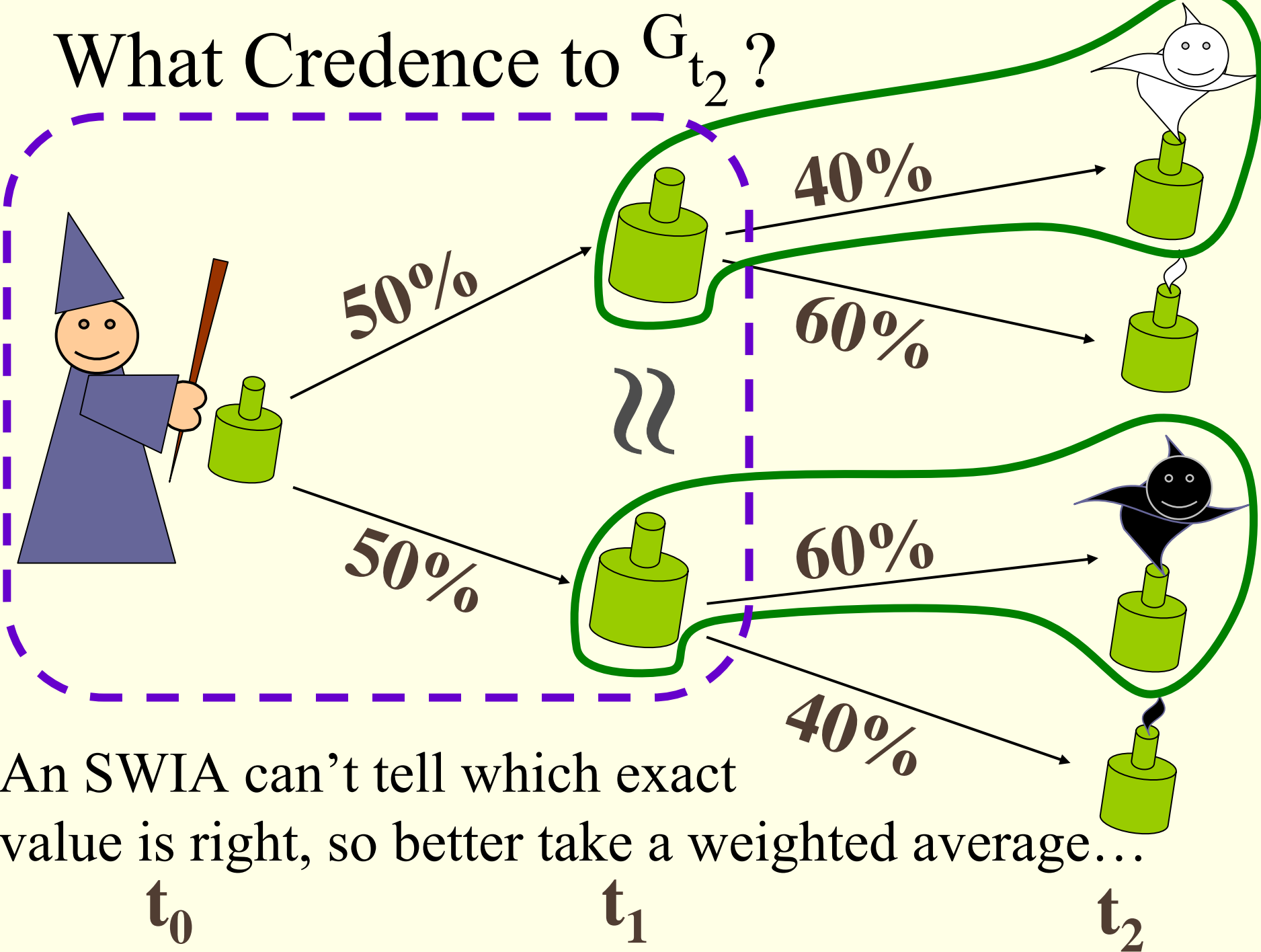
The Importance of PP

“A feature of Reality deserves the name of chance to the extent that it occupies the definitive role of chance; and occupying that role means obeying [PP]” (Lewis 1999, 245-6).

It would follow that, if we depart from PP, we'll risk losing any motivation for thinking that what we're talking about deserves the label 'chance'.

How great a departure must we make?

What Credence to G_{t_2} ?

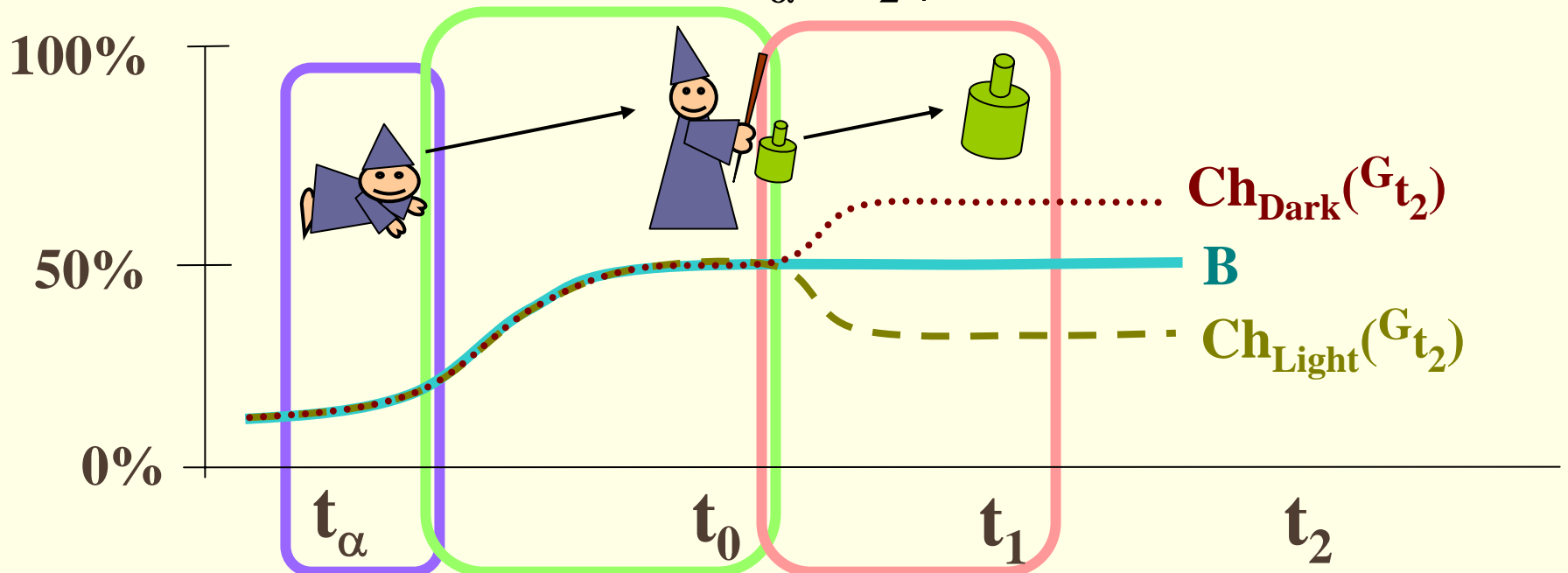


Three Options

- (1) Retreat to Lewis' original formulation of PP, which links chances only to credences *about chances* (not *laws* or *categorical history*).
- (2) Weaken PP to remain silent about (hopefully rare) cases involving free-floating chances.
- (3) Revise PP to say something positive about the relation between chances and credences about laws and categorical properties *in all cases* – formalize the intuitive idea that one's credence should be a weighted average of the potential values of a free-floating chance.

My Proposal

PP*: Given t_1 and G_{t_2} , let H be the full categorical history up to and including t_1 , and let t_α be a time prior to t_1 and prior to when the chances of G_{t_2} first float free, if there is such a time. At time t_1 , a suitably well-informed agent rationally must assign credence to G_{t_2} equal to the value of $\text{Ch}_{t_\alpha}(G_{t_2} | H)$ if it has a value.





8. Conclusion

All This is Neutral About...

...the ontological status of chances

- Reductionism: chance-talk is a fancy way of talking about patterns in categorical stuff.
- Non-reductionist realism about chances.

...the sort of connection between chances, laws and categorical properties.

- Lewis: CPs \Rightarrow (laws & chances)
- Maudlin: (CPs + laws) \Rightarrow chances
- Lange: (CPs + chances) \Rightarrow laws

Conclusions

- (1) Merlin's flourish-less enchantment should prompt us to revise or retreat from LMP, COND, and PP.
- (2) (Contra Lange) Such cases don't give any new reason to stake a particular stance regarding any of the difficult questions recently mentioned.
- (3) But they do help us to see more clearly what is at issue when we ask these difficult questions, and they help us avoid some deceptively alluring pitfalls as we seek answers.