Mutual Coupling Effect on Thermal Noise in Multi-antenna Wireless Communication Systems

Snezana Krusevac, Predrag B. Rapajic, Rodney A. Kennedy and Parastoo Sadeghi

***Abstract*— This paper presents a framework for the thermal noise analysis of mutually-coupled antennae in the multi-antenna system. The electromagnetic coupling for thermal noise is included in the analysis of the multi-antenna system with small antenna element spacings. The method for thermal noise power calculation for the multi-antenna system with coupled antennae is presented. The thermal noise behavior in the multi-antenna system is determined by applying the Nyquist's thermal noise theorem. The partial correlation of thermal noise for antenna spacing lower then a wavelength is confirmed. The signal-to-noise ratio (SNR) for the closely spaced antennae in the multi-antenna system is then estimated, using presented method for thermal noise analysis. Simulation results confirm that as the antenna spacing decreases to zero, the multi-antenna system starts to act like a single antenna system.**

***Index Terms*— Mutual Coupling, thermal noise, multi-antenna system**

I. INTRODUCTION

MULTIPLE-INPUT multiple output (MIMO) communication systems use antenna arrays to increase the communication capacity by exploiting the spatial properties of the multipath channel [1]. High capacity could be achieved by providing independence of the channel matrix coefficients, a condition generally achieved with wide antenna element spacings. But persistent miniaturization of subscriber units makes such separations impossible, and the resulting antenna mutual coupling [2] significantly impact the communication system performance.

The impact of antenna mutual coupling on the MIMO system has been evaluated by examining how the coupled antennae change the signal correlation [3]. The modifications in channel matrix coefficients are then used to assess mutual coupling effects on the system capacity [10], [4]. Additionally, the radiated power at the transmitter and the power collection capability due to effect of mutual coupling in the multi-antenna systems are presented in [11]. The effect of mutual coupling on the MIMO channel capacity through the signal-to-noise ratio was then presented in [6].

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The recent findings that signal-to-noise ratio are affected by mutual coupling, require an answer on a new issue about the thermal noise behaviour in the multi-antenna system. In [5], the noise originated from the amplifier at the receiver end of the MIMO system was included into consideration, but still not the thermal noise on the coupled antennae. The possibility that the thermal noise from radiated body could be induced in antenna was discussed in [7]. The fact that the partially correlated noises are introduced into receivers of two closely spaced antennae was discussed in [12].

Thermal noise correlation due to mutual coupling effects in the closely spaced antennae was a missing puzzle in order to asses the MIMO system communication performance with small antennae separation, especially critical for subscriber unit. The signal-to-noise ratio (SNR) is assessed for the multi-antenna system with large number of antenna placed in infinitesimal volume of space, which means that antenna spacing is almost zero. The calculation based on classical method, without considering mutual coupling effect, would give an infinite value for SNR. The result obtained by using classical method demonstrates the importance of the proper consideration of the coupled antennae in the multi-antenna system.

$$SNR = \lim_{[(n_R \to \infty) and (d \longrightarrow 0)]} \frac{n_R^2 P}{n_R N} \tag{1}$$

where d is the antenna spacing, P and N are the signal and noise power, respectively.

The aim of this paper is to provide the evidence that thermal noise is affected by mutual coupling effect. The model for theoretical elaboration of thermal noise in coupled multi-antenna system is presented. The application of Nyquist's thermal noise theorem [8] enables the disintegration of correlated part from the total thermal noise. The normalized correlated part of the thermal noise versus antenna spacings is then estimated. The error made by classical thermal noise consideration is compared with the presented method. The evidence that partially correlated noise appears for antenna spacing below a wavelength is provided.

Finally, we investigate the SNR behavior in the multi-antenna for antenna spacing below a wavelength, varying the number of dipoles from two to three. We confirm that as the antenna element spacings decrease to the almost overlapping case, the multi-antenna system acts like a single-antenna system with an equivalent radiated resistance.

The rest of this paper is organized as follows. Section II describes the multi-antenna system representation for the purpose of thermal noise analysis. In Section III, thermal noise coupling is elaborated. Thermal noise in two-dipole array and in the multi-antenna system are evaluated in sections IV-A and IV-B, respectively. Utilizing these results, the SNR is analyzed in section V. The concluding remarks are given at the end of the paper.

II. Multi-antenna System Representation

The multi-antenna system could be represented as a general linear network using a generalized form of Thevenin's theorem. The generalization of the Thevenin's theorem holds true not only for coherent sources but also for thermal noise sources [9]. It is valid even for the general linear network that may contain a number of inaccessible nodes together with internal voltage and current sources, whose location may be unknown. However, as long as there are only N independent accessible nodes, such a system is indistinguishable from a source free network, with the same impedance or admittance matrix, together with a set of N nodal current generators of infinite internal impedance. The current from the generator of the r^{th} node, in such equivalent network, is equal to the current flowing into the r^{th} node of the original network when all nodes of the latter are short-circuit to earth. The internal sources may be alternatively represented by set of N nodal voltage generators of infinite internal admittance such that the voltage across the generators in the r^{th} node is equal to the voltage across the r^{th} node of the original network when all the nodes of the latter are open-circuit. The nodal noise sources are not in general independent.

The multi-antenna system with $N = n_R$ antenna elements can be represented as a linear n_R-terminal-pair network containing internal prescribed signals or noise generators and it is specified completely with respect to its terminal pairs by its admittance matrix $\mathbf{Y}$ and a set of n_R nodal current generator $i_1, i_2, ..., i_{n_R}$.

In matrix form, $\mathbf{Y}$ denotes a squared matrix of order n_R

$$\mathbf{Y} = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n_R} \\ y_{21} & y_{22} & \cdots & y_{2n_R} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n_R 1} & y_{n_R 2} & \cdots & y_{n_R n_R} \end{pmatrix} \quad (2)$$

The complex amplitudes of thermal current generators are represented conveniently by a column vector $\mathbf{i}$:

$$\mathbf{i} = \begin{pmatrix} i_1 \\ i_2 \\ \cdot \\ i_{n_R} \end{pmatrix} \quad (3)$$

The nodal noise sources are not in general independent. The spectral density of the squared current can then be written in the matrix form.

$$\overline{\mathbf{i}\mathbf{i}^{\dagger}} = \begin{pmatrix} \overline{i_1 i_1^*} & \overline{i_1 i_2^*} & \cdots & \overline{i_1 i_{n_R}^*} \\ \overline{i_2 i_1^*} & \overline{i_2 i_2^*} & \cdots & \overline{i_2 i_{n_R}^*} \\ \cdots & \cdots & \cdots & \cdots \\ \overline{i_{n_R} i_1^*} & \overline{i_{n_R} i_2^*} & \cdots & \overline{i_{n_R} i_{n_R}^*} \end{pmatrix} \quad (4)$$

where the subscript † indicates the Hermitian transpose (complex conjugate transpose).

The internal sources may be alternatively represented by the set of n_R nodal voltage generators of infinite internal admittance such that r^{th} voltage across the generators in the r^{th} node is equal to the voltage across the r^{th} node of the original network when all the nodes of the latter are open-circuit.

III. Thermal Noise Correlation

Thermal noise originates from the body of antenna itself is self-noise or self-radiation. Besides, the self-radiated noise, induced thermal noise appears in antenna from radiated bodies in antenna encirclement [7]. The isolated receivers of two closely spaced antennae will receive partially correlated noise [12]. The magnitude correlation was calculated using a generalized form of Nyquist's thermal noise theorem given in [8]. It was shown that general nonreciprocal network with a system of internal thermal generators all at temperature T is equivalent to the source-free network together with a system of noise current generators I_r and I_s with infinite internal impedance [12]. Noise currents are correlated and their cross-correlation was given by:

$$\overline{I_s I_r} df = 2kT(Y_{sr} + Y_{sr}^*)df \quad (5)$$

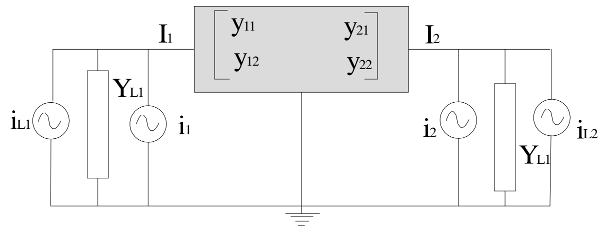
Alternatively the internal noise sources can be represented by a system of nodal voltage generators V_r and V_s, with zero internal impedance. The correlation of nodal voltage generators is given by:

$$\overline{V_s V_r} df = 2kT(Z_{sr} + Z_{sr}^*)df \quad (6)$$

where Z_{rs} and Y_{rs} are the mutual impedance and admittance respectively. Correlation is zero when the mutual coupling is purely reactive.

IV. Thermal noise power in the Multi-antenna system

The application of generalized Nyquist's thermal noise theorem allow us to determine thermal noise power of coupled antennae in the multi-antenna system. The theorem states that for passive network in thermal equilibrium it would be appear possible to represent the complete thermal-noise behaviour by applying Nyquist's theorem independently to each component element of network. The multi-antenna system with coupled antennae is represented by antenna self and mutual impedances. In order to determine thermal noise behaviour, self as well



1: Nodal network representation for two antenna array

as mutual impedances should be included into consideration. Thermal noise power calculation which accounts mutual coupling effects is given for two-antenna array. The generalization for the multi-antenna system is then made.

A. Two Antenna Array

The noise current system associated with the network itself is shown in Fig. 1 and for that case:

$$\begin{array}{l} I_1 = y_{11}V_1 + y_{12}V_2 = i_{L1} + i_1 - Y_{L1}V_1 \\ I_2 = y_{21}V_1 + y_{22}V_2 = i_{L2} + i_1 - Y_{L2}V_2 \end{array} \tag{7}$$

or

$$\begin{array}{l} i_{L1} + i_1 = (y_{11} + Y_{L1})V_1 + y_{12}V_2 \\ i_{L2} + i_1 = y_{21}V_1 + (y_{22} + Y_{L2})V_2 \end{array} \tag{8}$$

$$\begin{array}{l} i_{L1} + i_1 = (y_{11} + Y_{L1})V_1 + y_{12}V_2 \\ i_{L2} + i_1 = y_{21}V_1 + (y_{22} + Y_{L2})V_2 \end{array} \tag{9}$$

$$\begin{bmatrix} i_{L1} + i_1 \\ i_{L2} + i_1 \end{bmatrix} = \begin{pmatrix} y_{11} + Y_{L1} & y_{12} \\ y_{21} & y_{22} + Y_{L2} \end{pmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{pmatrix} y_{11} + Y_{L1} & y_{12} \\ y_{21} & y_{22} + Y_{L2} \end{pmatrix}^{-1} \begin{bmatrix} i_{L1} + i_1 \\ i_{L2} + i_1 \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{|D|} \begin{pmatrix} y_{22} + Y_{L2} & -y_{21} \\ -y_{12} & y_{11} + Y_{L1} \end{pmatrix} \begin{bmatrix} i_{L1} + i_1 \\ i_{L2} + i_1 \end{bmatrix} \tag{12}$$

$$V_1 = \frac{1}{|D|}(y_{22} + Y_{L2})(i_{L1} + i_1) - y_{21}(i_{L2} + i_1) \tag{13}$$

$$V_2 = \frac{1}{|D|}(y_{11} + Y_{L1})(i_{L2} + i_2) - y_{12}(i_{L1} + i_2) \tag{14}$$

where D is determinant of the following matrix

$$D = \begin{pmatrix} y_{11} + Y_{L1} & y_{12} \\ y_{21} & y_{22} + Y_{L2} \end{pmatrix} \tag{15}$$

The average power P_{L1} absorbed in the receiver load of the first antenna is proportional to

$$P_{L1} = \frac{1}{2}(Y_{L1} + Y_{L1}^*)\overline{V_1 V_1^*} \tag{16}$$

and similarly for the second antenna

$$P_{L2} = \frac{1}{2}(Y_{L2} + Y_{L2}^*)\overline{V_2 V_2^*} \tag{17}$$

Substituting the expression (13) in (29) yields

$$\begin{array}{l} P_{L1} = \dfrac{(Y_{L1} + Y_{L1}^*)}{2|D||D^*|} \\ ((y_{22} + Y_{L2})(y_{22}^* + Y_{L2}^*)(\overline{i_{L1} i_{L1}^*} + \overline{i_1 i_1^*}) \\ -y_{21}(y_{22}^* + Y_{L2}^*)\overline{i_1 i_2^*} \\ -y_{21}^*(y_{22} + Y_{L2})\overline{i_1^* i_2} \\ +y_{21} y_{21}^*(\overline{i_{L2} i_{L2}^*} + \overline{i_2 i_2^*})) \end{array} \tag{18}$$

Using the formula for nodal current correlation in (5), the final expression for thermal noise power absorbed in the receiver load of the first antenna becomes:

$$\begin{array}{l} P_{L1} = 2kT\dfrac{(Y_{L1} + Y_{L1}^*)}{2|D||D^*|} \\ ((y_{22} + Y_{L2})(y_{22}^* + Y_{L2}^*)((Y_{L1} + Y_{L1}^*) + (y_{11} + y_{11}^*)) \\ -\, y_{21}(y_{22}^* + Y_{L2}^*)(y_{12} + y_{12}^*) \\ -\, y_{21}^*(y_{22} + Y_{L2})(y_{12} + y_{12}^*) \\ +\, y_{21} y_{21}^*((Y_{L2} + Y_{L2}^*) + (y_{22} + y_{22}^*)) \end{array} \tag{19}$$

Similar expression can be obtained for second antenna:

$$\begin{array}{l} P_{L2} = 2kT\dfrac{(Y_{L2} + Y_{L2}^*)}{2|D||D^*|} \\ ((y_{11} + Y_{L1})(y_{11}^* + Y_{L1}^*)((Y_{L2} + Y_{L2}^*) + (y_{22} + y_{22}^*)) \\ -\, y_{12}(y_{11}^* + Y_{L1}^*)(y_{21} + y_{21}^*) \\ -\, y_{12}^*(y_{11} + Y_{L1})(y_{21} + y_{21}^*) \\ +\, y_{12} y_{12}^*((Y_{L1} + Y_{L1}^*) + (y_{11} + y_{11}^*)) \end{array} \tag{20}$$

The total noise for two coupled antenna elements, in frequency band B, can be obtained as a sum of these noise powers

$$N_{total} = \int_B P_{L1} df + \int_B P_{L2} df \tag{21}$$

B. Multi-antenna system

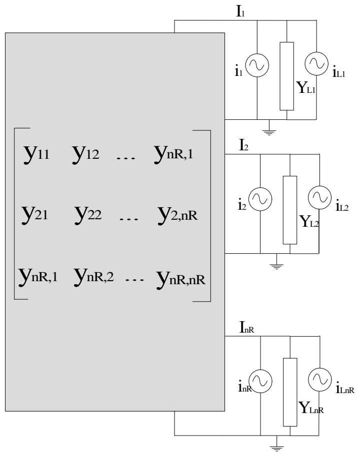
The nodal network representation for the multi-antenna system with n_R antenna elements is shown in Fig. 2.

$$\mathbf{I} = \begin{bmatrix} i_{L1} + i_1 \\ i_{L2} + i_1 \\ \cdot \\ i_{Ln_R} + i_{n_R} \end{bmatrix} \tag{22}$$

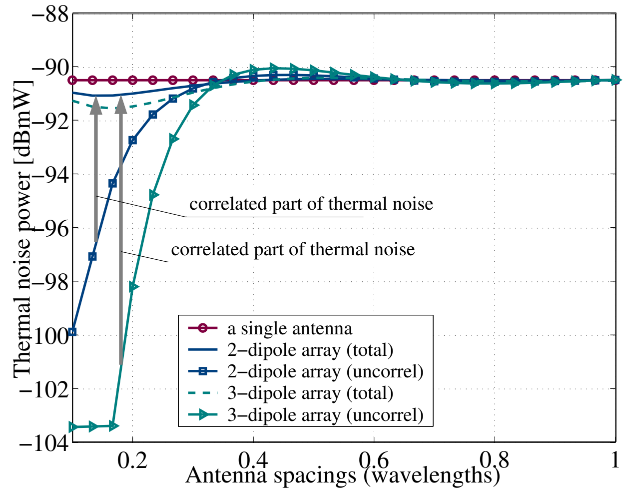
$$\mathbf{Y} + Y_L\mathbf{U} = \begin{pmatrix} y_{11} + Y_L & y_{12} & \cdot & y_{1n_R} \\ y_{21} & y_{22} + Y_L & \cdot & y_{2n_R} \\ \cdot & \cdot & \cdot & \cdot \\ y_{n_R 1} & y_{n_R 2} & \cdot & y_{n_R n_R} + Y_L \end{pmatrix} \tag{23}$$

$$\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ V_{n_R} \end{bmatrix} \tag{24}$$

From the Fig. 2, we can write the system equation as follows



2: Nodal network representation for multi-antenna system



3: Correlated part of thermal noise power versus antenna spacings for different number of dipoles in the antenna array

$$\mathbf{I} = \mathbf{i} + \mathbf{i}_L = (\mathbf{Y} + Y_L\mathbf{U})\mathbf{V} \tag{25}$$

or

$$\mathbf{V} = (\mathbf{Y} + Y_L\mathbf{U})^{-1}\mathbf{I} = (\mathbf{Y} + Y_L\mathbf{U})^{-1}(\mathbf{i} + \mathbf{i}_L) \tag{26}$$

$$\mathbf{N} = \frac{1}{2}(Y_L + Y_L^*)\overline{\mathbf{V}\mathbf{V}^\dagger} \tag{27}$$

$$\mathbf{V}\mathbf{V}^\dagger = (\mathbf{Y} + Y_L\mathbf{U})^{-1}\mathbf{I}\mathbf{I}^\dagger((\mathbf{Y} + Y_L\mathbf{U})^{-1})^\dagger \tag{28}$$

where $\dagger$ assign Hermitian transpose, $*$ assign complex conjugate

Now, we need to find

$$\mathbf{I}\mathbf{I}^\dagger = (\mathbf{i} + \mathbf{i}_L) \times (\mathbf{i} + \mathbf{i}_L)^\dagger \tag{29}$$

Based on (5) the following relations are valid

$$\begin{aligned} 1.\quad \overline{i_j i_k^*}df &= 2kT(y_{jk} + y_{jk}^*); \\ 2.\quad \overline{i_{Lj} i_k^*} &= 0; \\ 3.\quad \overline{i_{Lj} i_{Lk}^*} &= 0\ , j \neq k; \end{aligned} \tag{31}$$

The squared of currents is given in (30) and (32), given in the next page

and

The final expression for squared of currents is given in (33)

$$\mathbf{I}\mathbf{I}^\dagger = 2kT(\mathbf{Y} + \mathbf{Y}^* + (Y_L + Y_L)^*\mathbf{U}) \tag{34}$$

Substituting $Y_L + Y_L^* = 2G_L$ and $\mathbf{Y}_a = \mathbf{Y} + Y_L\mathbf{U}$, we can finally write

$$\mathbf{N} = 2kTG_L[\mathbf{Y}_a^{-1} \times (\mathbf{Y}_a + \mathbf{Y}_a^*) \times (\mathbf{Y}_a^{-1})^\dagger] \tag{37}$$

or

$$\mathbf{N} = 2kTG_L[\mathbf{Y}_a^{-1} \times Real(\mathbf{Y}_a) \times (\mathbf{Y}_a^{-1})^\dagger] \tag{38}$$

$$N_{total} = 2kTG_L \int_B \mathrm{trace}([\mathbf{Y}_a^{-1} \times Real(\mathbf{Y}_a) \times (\mathbf{Y}_a^{-1})^\dagger])df \tag{39}$$

where operator trace gives the sum of diagonal matrix elements and B is the frequency band of receiver bandpass filter.

The total thermal noise power received from the antenna array in the receiver load is given in (39). In this paper, we provide the thermal noise power calculation received from the coupled antennae. The thermal noise received in one antenna consists of two parts, its own thermal noise and induced thermal noise from the adjacent antenna elements.

V. SNR ANALYSIS

The output SNR of the multi-antenna system is the most commonly accepted measure of its communication performance. The analysis in previous sections indicates on the importance of including the mutual coupling effect on thermal noise. The received thermal noise power from the closely spaced antenna elements in multi-antenna system is given in (39).

The method provides accurate calculation of the output SNR in the multi-antenna system, even for antenna spacings below a wavelength. The presented method considers the influence of the mutual coupling effect on both signal and noise.

The fair comparison between the antenna arrays with different number of antenna elements in terms of the

$$\mathbf{I}\mathbf{I}^{\dagger} = \begin{bmatrix} i_{L1}+i_1 \\ i_{L2}+i_1 \\ \cdot \\ i_{Ln_R}+i_{n_R} \end{bmatrix} \times \begin{bmatrix} i_{L1}^*+i_1^* & i_{L2}^*+i_1^* & \cdots & i_{Ln_R}^*+i_{n_R}^* \end{bmatrix} \tag{30}$$

$$\mathbf{I}\mathbf{I}^{\dagger} = \begin{pmatrix} i_1 i_1^* + i_{L1} i_{L1}^* & i_1 i_2^* & \cdots & i_1 i_{n_R}^* \\ i_2 i_1^* & i_2 i_2^* + i_{L2} i_{L2}^* & \cdots & i_2 i_{n_R}^* \\ \cdots & \cdots & \cdots & \cdots \\ i_{n_R} i_1^* & i_{n_R} i_2^* & \cdots & i_{n_R} i_{n_R}^* + i_{Ln_R} i_{Ln_R}^* \end{pmatrix} \tag{32}$$

$$\mathbf{I}\mathbf{I}^{\dagger} = 2kT \begin{pmatrix} y_{11}+y_{11}^*+Y_L+Y_L^* & y_{12}+y_{12}^* & \cdots & y_{1n_R}+y_{1n_R}^* \\ y_{21}+y_{21}^* & y_{22}+y_{22}^*+Y_L+Y_L^* & \cdots & y_{2n_R}+y_{2n_R}^* \\ \cdots & \cdots & \cdots & \cdots \\ y_{n_R 1}+y_{n_R 1}^* & y_{n_R 2}+y_{n_R 2}^* & \cdots & y_{n_R n_R}+y_{n_R n_R}^*+Y_L+Y_L^* \end{pmatrix} \tag{33}$$

$$\mathbf{V}\mathbf{V}^{\dagger} = 2kT(\mathbf{Y}+Y_L\mathbf{U})^{-1}(\mathbf{Y}+\mathbf{Y}^*+(Y_L+Y_L)^*\mathbf{U})((\mathbf{Y}+Y_L\mathbf{U})^{-1})^{\dagger} \tag{35}$$

$$\mathbf{N} = kT(Y_L+Y_L^*)(\mathbf{Y}+Y_L\mathbf{U})^{-1}(\mathbf{Y}+\mathbf{Y}^*+(Y_L+Y_L)^*\mathbf{U})((\mathbf{Y}+Y_L\mathbf{U})^{-1})^{\dagger} \tag{36}$$

signal-to-noise ratio could be obtained only by assuming that incident fields have the same powers. The analysis of the SNR will be done under the elemental power constraint. The incident field at each antenna element is limited. The incident field is $E = V_{sig} * h$, where V and h are induced voltage in the antenna elements and the height of antenna elements, respectively. In matrix form, we can write:

$$\mathbf{I}_{sig} = (\mathbf{Y}+Y_L\mathbf{U})\mathbf{V}_{sig} \tag{40}$$

where, $\mathbf{I}_{sig}$ and $\mathbf{V}_{sig}$ are current and voltage column vector.

$$\mathbf{P}_s = \frac{R_L}{2}\mathbf{I}_{sig}\mathbf{I}_{sig}^{\dagger} \tag{41}$$

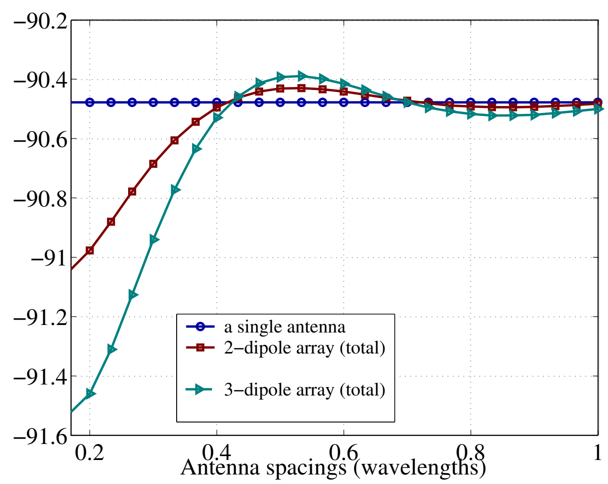
Finally, the SNR is:

$$SNR = \frac{\mathrm{trace}(\mathbf{P}_s)}{N_{total}} \tag{42}$$

VI. Simulation Results

To demonstrate the application of the analysis framework developed in this paper and to illustrate the impact of the noise coupling on MIMO system, we use the a model problem consisting of two and three half-wave dipoles in multi-antenna system. The simplicity of this problem allows us to accurately characterize coupled antennae and draw basic conclusions concerning the system operation.

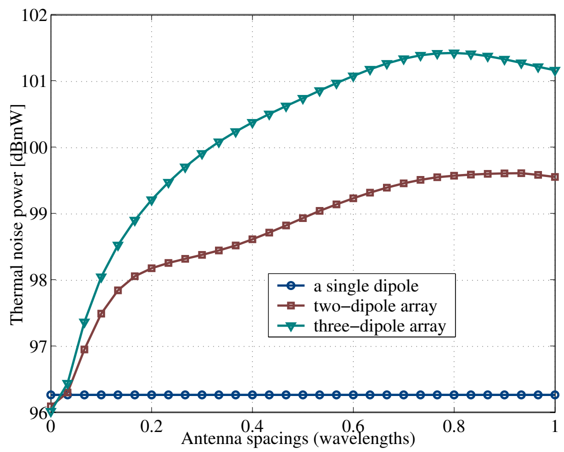
The existence of correlated part of thermal noise is shown in Fig. 3. The disintegration of correlated part from total thermal noise of one dipole in dipole array become possible by applying Nyquist's thermal theorem. Thus, thermal noise that originates from self-impedance



4: The thermal noise power of one dipole in antenna array versus antenna spacings

is assigned as uncorrelated, as it comes from its own antenna elements. While, thermal noise that originates from all mutual impedances represents correlated part of thermal noise. Correlated part of thermal noise is actually induced thermal noise from adjacent antennae. The correlated part grows as antenna separation decreases. The correlated part of thermal noise is higher for the same antenna spacing when the number of antenna element in the multi-antenna system increases, as the number of sources grows.

Fig. 4 plots the total thermal noise power of one dipole in the antenna array. Results are given for one dipole in two- and three-dipole array and for one isolated dipole. We first observe that for close antenna spacings with high coupling, the thermal noise power of one dipole



5: Signal-to-noise ratio versus antenna element spacings for different number of dipoles in antenna array

in two- and three-dipole array decreases. For two-dipole array, thermal noise power drops for 0.5dB. The drop is almost 1dB for three-dipole array. For distance greater then 0.4λ, the curves merge into one another. This is a expected result as the influence of the mutual coupling gradually decreases.

Fig. 5 demonstrates the combined effects of electromagnetic coupling for signal and thermal noise on the SNR in the multi-antenna system. Here, the SNR is computed for one dipole, for two-dipole array and for three-dipole array. The total signal and thermal noise power received from the multi-antenna system are taken into account. From Fig. 5, we confirm that as antennae approach to each other, antennae in multi-antenna system start to act as a single antenna with an equivalent resistance. Physical argument suggest that, but obtaining this result from simulation is now enabled by considering mutual coupling for thermal noise. The decline of the SNR for the two- and three-antenna array is additionally supported by the decrease of the thermal noise power shown in Fig. 4.

VII. Conclusion

This paper has outlined a procedure for analyzing the thermal noise in the multi-antenna system with coupled antennae. The mutual coupling is included for thermal noise consideration in the multi-antenna system with small antenna spacings. The method for thermal noise power calculation in the multi-antenna system is presented, valid even for antenna spacing lower then a half of wavelength. The method accurately depicts thermal noise behavior of the multi-antenna system with small antenna spacing. The partial correlation of thermal noise for antenna spacing below a wavelength is confirmed. The SNR behavior in multi-antenna system for small antenna elements is analyzed. Simulation results confirm that the multi-antenna system start to act as a single antenna with an equivalent radiated resistance when antenna spacings drop to almost zero. The result is intuitive, but the accurate results could only be obtained by considering the electromagnetic coupling for both signal and noise. We show that the mutual coupling effect has a considerable impact on the MIMO system performance.

References

[1] G.J. Foschini, M.J. Gans, “On Limits of Wireless Communication in Fading Environment when Using Multiple Antennas”, *Wireless Personal Communication*, vol.6, pp. 311-335, March 1998.

[2] C. Balanish, *Antenna Theory: Analysis and Design*, Inc, New York: Wiley, 1997.

[3] I.J. Guptha, A.K. Ksienski,“Effect of the Mutual coupling on the Performance Of the Adaptive Arrays”, *IEEE Trans. On Antennas and Propagation*, 31(5):785-791, September 1983,

[4] R. Janaswamy, “Effect of element mutual coupling on the capacity of fixed length lenar arrays”, :*IEEE Antennas Wireless Propagat. Lett.*, vol. 1, pp. 157-160, 2002.

[5] M. Morris, M. Jensen, “Network model for MIMO systems with coupled antennas and noisy amplifiers”, :*IEEE Trans. Antennas Propagat.*, 2002.

[6] S. Krusevac, P. Rapajic, R. Kennedy, “Method for MIMO Channel Capacity Estimation for Electro-magnetically Coupled transmit antenna elements”, in *AusCTW 2004*, Feb. 2004., pp. 122-126

[7] S.M. Rytov, Yu.A. Krastov,V.I. Tatarskii,*Principles of Statistical Radiophysics 3, Elements of the Random Fields* (Springer, Berlin, Heidenberg, New York 1987)

[8] E.J.Schremp,MIT Rad. Lab. Series 18, Valley and Wallman, *Vacuum Tube Amplifier* (McGraw-Hill, Inc.,)NY, 1949, chapter 12

[9] A.T. Starr,*Electric Circuit and Wave Filters*, Pitman, London, 1946, second edition, p. 78.

[10] T. Sventenson, A. Ranheim “Mutual coupling effects on the capacity of the multielement antenna system”, *Acoustic, Speech and Signal Processing* ,2001, Proceedings(ICASSP’01)

[11] J.W. Wallace, M.A. Jensen, “Mutual coupling in MIMO Wireless Systems: A rigorous network theory analysis”,*IEEE Trans. Wireless Commun.*, vol. 3, pp. 1317-1325, 2004

[12] R.Q. Twiss, “Nyquist’s and Thevenin’s Generalized for Nonreciprocal Linear networks”,*J. Applied Phys.* ,vol. 26, pp. 559-602,May, 1955