

The Effect of Memory Order on the Capacity of Finite-State Markov and Flat-Fading Channels

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Abstract—In this paper, we study the effect of memory order on the capacity of finite-state Markov channels (FSMC). We analytically compare the capacity of an originally high-order FSMC model with the capacity of its reduced memory order version. We show that the capacity difference is caused by two factors: 1) the channel entropy difference, and 2) the channel observability difference between the two models. While the first factor, alone, results in underestimation of the original FSMC capacity by the reduced-order FSMC model, due to the existence of the second factor, capacity overestimation can also occur. Explicit examples of FSMC models are provided, where the reduced-order FSMC model overestimates the capacity of the original high-order channel. To show the practical significance of the analysis, we model time-varying flat-fading (FF) channels with FSMC models. It is observed that the first-order FSMC models can provide both higher and lower estimates of the FF channel capacity, compared to higher order FSMC models.

I. INTRODUCTION

Time-varying flat-fading (FF) channels are commonly found in mobile communication systems [1]. Computing the information capacity of the correlated and time-varying FF channel in Clarke's model [1], with no channel state information (CSI) at the transmitter and at the receiver, is an open problem [2]. Therefore, the use of alternative channel models for the capacity analysis of time-varying FF channels is often inevitable. Evidently, inaccuracy in the simplified time-varying channel model can potentially result in inaccurate estimates of the channel's true information capacity.

A widely-used model for time-varying FF channels is the finite-state Markov channel (FSMC) [2]–[5]. One of the advantages of using FSMC models is that the information capacity (or at least the information rate) of the FSMC model may be computed [3], [6]–[8]. However, the memory order of the underlying Markov chain in an FSMC model exponentially increases the number of channel states. This, in turn, increases the computational complexity of the FSMC model, including its capacity analysis. Therefore, low-order FSMC models (in particular, first-order FSMC models) are preferred over high-order FSMC models. The wide-spread use of first-order FSMC models for FF channels is despite the reported results in [9], which indicate the inaccuracy of the first-order FSMC modeling of FF channels in medium-speed fading conditions. The capacity of time-varying FF channels has been analyzed in [5] with the use of first-order FSMC models. Nevertheless,

the reliability of the capacity analysis using low-order FSMC models to provide accurate capacity estimates of the original time-varying channel deserves further consideration.

In this paper, we study the effect of memory order on the capacity of FSMC models. For the theoretical analysis, we assume that an original M^{th} -order FSMC model is simplified to a lower order FSMC model (to reduce the computational complexity). The objective is to find how the capacities of the original and reduced-order FSMC models compare. We study the practical class of FSMC models, for which reducing the channel memory order is possible without affecting the channel observation law (FSMC models with memoryless observation property (MOP)). For example, in FF channels, the channel observation law only depends on the latest FF channel gain, regardless of the previous FF channel gains. Hence, when FF channels are mapped into M^{th} -order FSMC models, the resulting FSMC model possesses MOP.

It is known that if a *directly observable* M^{th} -order Markov chain is approximated with an $(M-1)^{\text{th}}$ -order Markov chain, the approximate Markov chain is closer to a memoryless process than the original chain. Moreover, the entropy rate is monotonically increasing with reducing the Markov chain memory order. Likewise, it is highly expected that the reduced-order FSMC model *monotonically* underestimates the true capacity of the high-order FSMC model. However, the fundamental difference between Markov chains and FSMC models is that the Markov process in the FSMC model is *not directly observable* or is *hidden* in the channel observations. The effect of this fundamental difference on the capacity of FSMC-MOP models is analytically investigated in the present work.

The contributions of the paper are summarized as follows.

- 1) In Section III-A, we quantitatively identify two factors that contribute to the capacity difference between the reduced-order and original FSMC models: a) *the channel entropy rate difference*, and b) *the channel observability difference* between the two models. While the first factor, alone, results in capacity underestimation by the reduced order FSMC model, due to the existence of the second factor, the reduced-order FSMC model might overestimate the true channel capacity.
- 2) In Section III-B, we provide explicit examples of FSMC models, where the true high-order FSMC capacity is

overestimated by the reduced-order FSMC model.

- 3) In Section IV, we show the practical applications of the study by analyzing the capacity of time-varying FF channels using M^{th} -order FSMC models. This is an extension of our previous work in [5] from the first-order to higher order FSMC modeling of FF channels. The analysis shows the non-monotonic behavior of the FSMC capacity to the assumed FF channel memory order. In low SNR conditions, the first-order FSMC model for the FF channel phase provides higher estimates of the channel capacity than higher order FSMC models.

II. SYSTEM MODEL

A. The M^{th} -order FSMC Model

A stationary and irreducible M^{th} -order Markov chain [6] with a finite state space is considered to be the underlying FSMC state process. The FSMC state at time index k is denoted by s_k . For $M > 1$, FSMC states are composite states that consist of M smaller elements called FSMC *substates*. The FSMC substate at time index k is denoted by r_k . The number of FSMC substates is L , where $r_k \in \mathcal{A} = \{0, \dots, L-1\}$. Hence, the M^{th} -order FSMC model has $N = L^M$ states, which belong to the set $\mathcal{S} = \mathcal{A}^M$. The FSMC states are ordered as M -tuples in base L , starting from all zero state $(0, \dots, 0)_L$ and ending in $(L-1, \dots, L-1)_L$. That is,

$$s_k = (r_k, \dots, r_{k-M+2}, r_{k-M+1}). \quad (1)$$

The FSMC state transition probability matrix is denoted by \mathbf{P} with elements

$$P_{s' \rightarrow s} \triangleq \Pr(S_k = s | S_{k-1} = s'), \quad s, s' \in \mathcal{S}. \quad (2)$$

The FSMC input at time index k is denoted by $x_k \in \mathcal{X} = \{0, \dots, Q-1\}$, where Q is the number of channel input levels. It is assumed that the FSMC model is data-independent, i.e. FSMC states evolve independently of the channel input process. The FSMC output at time index k is denoted by $y_k \in \mathcal{Y} = \{0, \dots, U-1\}$, where U is the number of channel output levels. Each FSMC state is associated with a discrete memoryless channel (DMC). The channel observation law is the conditional probability mass function (pmf) of the channel output y_k given the channel input x_k and the channel state s_k and is denoted by

$$\Pr(Y_k = y | X_k = x, S_k = s), \quad x \in \mathcal{X}, y \in \mathcal{Y}, s \in \mathcal{S}. \quad (3)$$

If in an FSMC model, there exists a deterministic function of the channel input and output process $z \triangleq g(y, x)$, such that the channel observation law in (3) is simplified to

$$\Pr(Z_k = g(y, x) = z | S_k = s), \quad (4)$$

a variable-noise FSMC model is obtained [3]. Throughout the paper, we study variable-noise FSMC models.

In the FSMC model, a channel observation is made for every transition of FSMC state s_{k-1} to another FSMC state s_k . According to (1), the state transition in an M^{th} -order FSMC model is due to a new FSMC substate r_k . If the channel observation law is a probabilistic function of the latest

FSMC substate, regardless of the previous FSMC substates, an FSMC model with memoryless observation property (MOP) is obtained, which may be referred to as the FSMC-MOP. In this case, the channel observation law in (4) is simplified to

$$\begin{aligned} \Pr(Z_k = z | S_k = s) &= \\ \Pr(Z_k = z | R_k = r, \dots, R_{k-M+1} = r') &= \\ \Pr(Z_k = z | R_k = r). \end{aligned} \quad (5)$$

In FSMC-MOP models, channel states that have the same current substate, possess the same channel observation law. FSMC-MOP models have a strong practical foundation. For example, in FF channels, the channel observation law only depends on the latest FF channel gain (refer to (19)). Therefore, in the mapping of FF channels into M^{th} -order FSMC models, FSMC-MOP models are obtained.

B. Memory Order Reduction of FSMC Models with Memoryless Observation Property

When the FSMC model possesses MOP, it is possible to reduce (approximate) the memory order of the underlying Markov chain without affecting the channel observation law. For FSMC memory orders $M > 1$, $(M-1)^{\text{th}}$ -order approximation of the FSMC-MOP model is possible by ignoring the dependence of states on the oldest FSMC substate r_{k-M+1} , or equivalently by merging states that differ only in the oldest substate r_{k-M+1} . In the $(M-1)^{\text{th}}$ -order FSMC model, channel state at time index k is denoted by q_k and is written as

$$q_k \triangleq (r_k, \dots, r_{k-M+2}). \quad (6)$$

For the state q_k in the $(M-1)^{\text{th}}$ -order FSMC model, the original state in the M^{th} -order FSMC model may be one of the following states

$$\begin{aligned} s_k &\triangleq (q_k, r_{k-M+1}) \\ &= (r_k, \dots, r_{k-M+2}, r_{k-M+1}), \quad \forall r_{k-M+1} \in \mathcal{A}. \end{aligned} \quad (7)$$

The main parameter of the reduced-order FSMC model is its state transition probability, which is denoted by

$$\begin{aligned} P_{q' \rightarrow q} &\triangleq \Pr(Q_k = q | Q_{k-1} = q') \\ &= \frac{\Pr(Q_k = q, Q_{k-1} = q')}{\sum_{\forall q} \Pr(Q_k = q, Q_{k-1} = q')}. \end{aligned} \quad (8)$$

Using (7), (8) can be written as

$$P_{q' \rightarrow q} = \frac{\Pr(S_k = (q, r_{k-M+1}))}{\sum_{\forall q} \Pr(S_k = (q, r_{k-M+1}))}. \quad (9)$$

Therefore, the state transition probability in the reduced-order FSMC model can be easily obtained from the stationary state probabilities of the original M^{th} -order FSMC model. The stationary state probability vector in the M^{th} -order FSMC is denoted by π , which is obtained by solving the eigenvector equation $\mathbf{P}^T \pi = \pi$ (superscript T denotes matrix transpose).

Example 1: A second-order FSMC model with four states is shown in Fig. 1-(a). Each FSMC state composes of one current and one previous FSMC substate from the binary

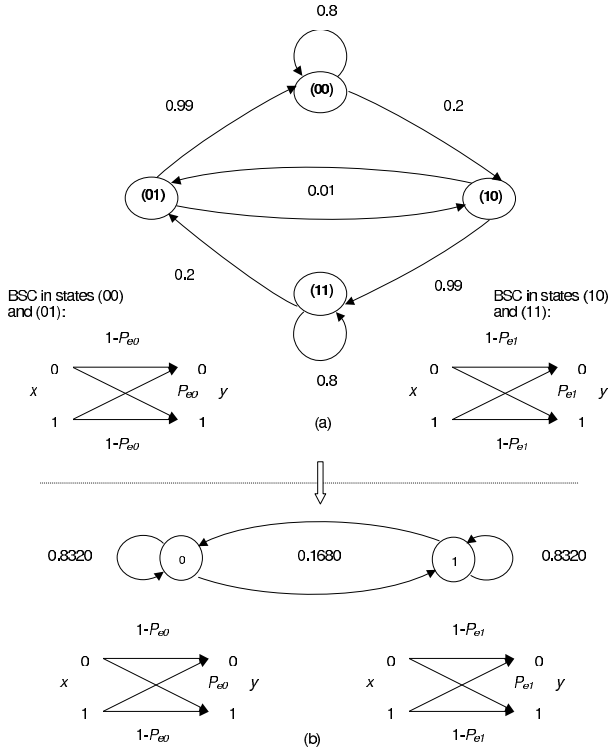


Fig. 1. (a) A second-order FSMC-MOP model. (b) The first-order approximation of the FSMC-MOP model in (a).

field $r \in \mathcal{A} = \{0, 1\}$. The channel in each FSMC state is a binary symmetric channel (BSC). It is observed from Fig. 1-(a) that states $s = (00)$ and $s = (01)$ have the same channel observation law (since the current FSMC substate is 0 in both states), which is given as $\Pr(Z = 1|S = (00)) = \Pr(Z = 1|S = (01)) = P_{e0}$. The channel observation law in states $s = (10)$ and $s = (11)$ is the same (since the current FSMC substate is 1 in both states) and is given as $\Pr(Z = 1|S = (10)) = \Pr(Z = 1|S = (11)) = P_{e1}$. Therefore, according to the definition in (5), the FSMC model possesses MOP and it is possible to reduce the second-order FSMC model to a first-order FSMC. This approximation does not change the channel observation law in the reduced states. This is shown in Fig. 1-(b). State $q = 0$ in the new model corresponds to the merging of states $s = (00)$ and $s = (01)$ in Fig. 1-(a). State $q = 1$ in the new model corresponds to the merging of states $s = (10)$ and $s = (11)$ in Fig. 1-(a). According to (9),

$$\begin{aligned} \Pr(Q_k = 0|Q_{k-1} = 0) &= \frac{\Pr(Q_k = 0, Q_{k-1} = 0)}{\sum_{q=0}^1 \Pr(Q_k = q, Q_{k-1} = 0)} \\ &= \frac{\Pr(S = (00))}{\Pr(S = (00)) + \Pr(S = (10))} \\ &= \frac{0.4160}{0.4160 + 0.0840} = 0.8320 \quad \square \end{aligned}$$

The FSMC state entropy rate is affected by the low memory order approximation. Let $\mathcal{H}(S)$ and $\mathcal{H}(Q)$ denote the state entropy rates in the M^{th} -order and $(M-1)^{\text{th}}$ -order FSMC

models, respectively. Using the (1) and (6), it is shown that

$$\begin{aligned} \mathcal{H}(S) &= H(R_k|R_{k-1}, \dots, R_{k-M}) \\ &\leq H(R_k|R_{k-1}, \dots, R_{k-M+1}) = \mathcal{H}(Q), \end{aligned} \quad (10)$$

where the inequality in (10) follows from the fact that conditioning reduces entropy. Therefore, the state process in the reduced-order FSMC model has always an entropy rate greater than or equal to the state entropy rate in the original FSMC model, making it a less predictable stochastic process.

III. THE EFFECT OF MEMORY ORDER ON THE FSMC CAPACITY

A. Theoretical Analysis

In order to explain the effect of channel memory order on the capacity of FSMC models, it is assumed that the FSMC model is uniformly-symmetric and variable-noise [3]. The capacity of a uniformly-symmetric and variable-noise M^{th} -order FSMC model is written as [3]

$$C^{(M)} = \log U - \mathcal{H}^{(M)}(Z), \quad (11)$$

where U is the number of channel output levels and $\mathcal{H}^{(M)}(Z)$ is the FSMC error entropy rate. Similarly, the capacity of $(M-1)^{\text{th}}$ -order approximate FSMC model is given as

$$C^{(M-1)} = \log U - \mathcal{H}^{(M-1)}(Z), \quad (12)$$

where $\mathcal{H}^{(M-1)}(Z)$ is the FSMC error entropy rate in the reduced-order FSMC model. From (11) and (12), it becomes clear that the capacity difference between the reduced-order and original FSMC models is

$$\Delta C \triangleq C^{(M-1)} - C^{(M)} = \mathcal{H}^{(M)}(Z) - \mathcal{H}^{(M-1)}(Z). \quad (13)$$

We use the definition of the mutual information rate between FSMC states and FSMC error process to write

$$\begin{aligned} \mathcal{I}^{(M)}(Z; S) &= \mathcal{H}^{(M)}(Z) - \mathcal{H}(Z|S) \\ &= \mathcal{H}(S) - \mathcal{H}(S|Z). \end{aligned} \quad (14)$$

Using (14), the FSMC error entropy rate in the M^{th} -order FSMC model can be written as

$$\mathcal{H}^{(M)}(Z) = \mathcal{H}(S) - \mathcal{H}(S|Z) + \mathcal{H}(Z|S). \quad (15)$$

Similarly, the FSMC error entropy rate in the $(M-1)^{\text{th}}$ -order FSMC model is

$$\mathcal{H}^{(M-1)}(Z) = \mathcal{H}(Q) - \mathcal{H}(Q|Z) + \mathcal{H}(Z|Q). \quad (16)$$

The last term in the right hand side of (15) and (16) is the FSMC error entropy rate with perfect CSI. In FSMC-MOP models, the FSMC error process Z given the FSMC state process S (or Q) only depends on the latest FSMC substate R (refer to (5)). Therefore, we can write

$$\mathcal{H}(Z|S) = \mathcal{H}(Z|Q) = \mathcal{H}(Z|R). \quad (17)$$

In other words, the FSMC memory order does not affect the CSI entropy rate of the error process in FSMC-MOP models. From (13), (15), (16), and (17), it is concluded that

$$\Delta C = \underbrace{\mathcal{H}(S) - \mathcal{H}(Q)}_{\Delta H} - \underbrace{[\mathcal{H}(S|Z) - \mathcal{H}(Q|Z)]}_{\Delta H Z}. \quad (18)$$

We refer to ΔH as the *channel (state) entropy rate difference* between the reduced-order and original FSMC models. It was shown in (10) that $\Delta H = \mathcal{H}(S) - \mathcal{H}(Q) \leq 0$, which by itself predicts capacity underestimation ($\Delta C \leq 0$). However, due to the existence of $\Delta HZ = \mathcal{H}(S|Z) - \mathcal{H}(Q|Z)$ in (18), the effect of FSMC memory order on the capacity cannot be generally predicted without direct evaluation of (18). We refer to ΔHZ as the *channel (state) observability difference* between the reduced-order and original FSMC models in the presence of channel error process Z , which is measured by the entropy (uncertainty) of the FSMC state process given the FSMC error process.

B. Numerical Analysis

Fig. 2 shows the capacity of the original and reduced-order FSMC models, with the same structure as those shown in Fig. 1, for a range of BSC crossover probabilities $P_{e0} = 1 - P_{e1}$ and two different FSMC state transition probabilities. It is observed that for the less persistent FSMC model (Ch1), when the BSCs are noisy (large P_{e0}), the reduced-order FSMC model overestimates the original FSMC capacity. As the BSCs become less noisy, the reduced-order FSMC model underestimates the original FSMC capacity. In Ch2, which is a more persistent FSMC model than Ch1, it is observed that the reduced-order FSMC overestimates the original FSMC capacity for a wider range of BSC crossover probabilities P_{e0} .

Next, we study the effect of memory order on the FSMC capacity for various state transition probabilities. The FSMC models have the same structure as those shown in Fig. 1. The results are shown in Fig. 3, where the parameter is $x \triangleq \Pr(S_k = (00)|S_{k-1} = (00)) = \Pr(S_k = (11)|S_{k-1} = (11))$. The BSC crossover probability is $P_{e0} = 1 - P_{e1} = 0.05$ in the two upper graphs and $P_{e0} = 1 - P_{e1} = 0.2$ in the two lower graphs. It is clearly observed that in the noisier case ($P_{e0} = 1 - P_{e1} = 0.2$), the reduced-order FSMC model is overestimating the original FSMC capacity. However, in the less noisy case ($P_{e0} = 1 - P_{e1} = 0.05$), capacity overestimation occurs only for more persistent FSMC models.

The results in Fig. 2 and Fig. 3 may be interpreted using (13). When the FSMC states are highly observable in the presence of FSMC errors ($\mathcal{H}(S|Z) \approx \mathcal{H}(Q|Z) \approx 0$), the reduced-order FSMC model underestimates the original FSMC capacity, especially when ΔH is large. For the studied FSMC examples, this happens when $P_{e0} = 1 - P_{e1}$, $P_{e0} \rightarrow 0$, and the original FSMC model is relatively fast.

IV. THE EFFECT OF MEMORY ORDER ON THE CAPACITY OF FLAT-FADING CHANNELS

In FF channels, the received signal y_k is defined as [1]

$$y_k = c_k x_k + n_k = a_k e^{j\theta_k} x_k + n_k, \quad (19)$$

where x_k is the transmitted symbol with symbol energy \mathcal{E}_s , n_k is the complex-valued, additive white Gaussian noise (AWGN) with variance per dimension $N_0/2$, and c_k is the complex-valued Gaussian FF channel gain with a normalized power equal to 1. The actual realization of the FF gain c_k in (19) is often unknown and varies with time. We assume that the

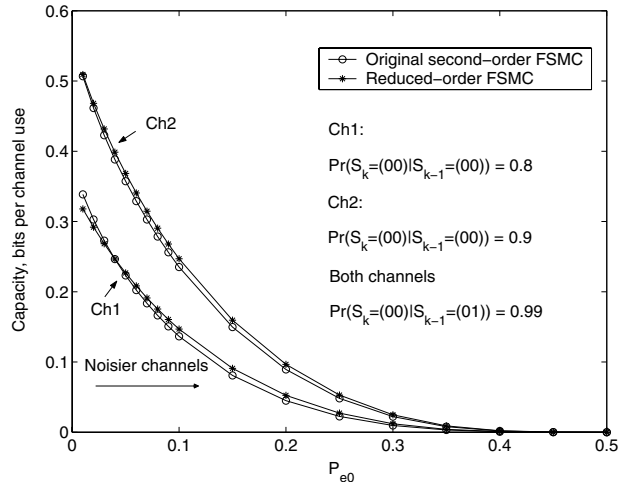


Fig. 2. The effect of memory order on the capacity of FSMC models with the same structure as those shown in Fig. 1. The varying parameter is the BSC crossover probability $P_{e0} = 1 - P_{e1}$. Except for the least noisy cases in the faster channel (Ch1), the first-order FSMC model overestimates the second-order FSMC capacity.

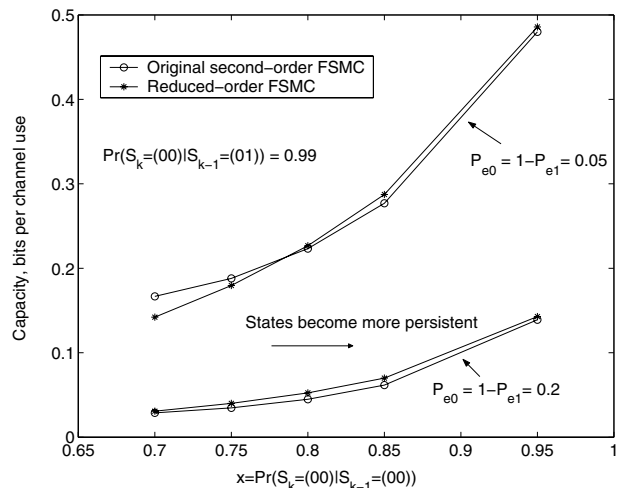


Fig. 3. The effect of memory order on the capacity of FSMC models with the same structure as those shown in Fig. 1. The varying parameter is the FSMC state persistence in state $s = (00)$. In the noisier channel, capacity overestimation by the first-order FSMC model occurs for all considered FSMC states persistencies. In the less noisy channel, capacity overestimation occurs only for more persistent FSMC models.

normalized auto-correlation function (ACF) of the FF channel gain in each real and imaginary dimension is defined as $\rho_m = 0.5J_0(2\pi m f_D T)$ [1], where J_0 is the zero-order Bessel function of the first kind. $f_D T$ is the normalized fading rate.

Computing the information capacity of the correlated and time-varying FF channel in (19), where no CSI is available at the receiver side is still unsolved [2]. A proposed approach is to model the FF channel with an M^{th} -order FSMC model and obtain an estimate of the channel capacity using the resulting FSMC model [3], [5]. In this approach, the FF channel gain is partitioned into L non-overlapping intervals or substates. A vector of the current and $M - 1$ previous FF channel gain substates form the FSMC states. Since the

channel observation law in the original FF channel in (19) is independent of previous FF channel gains, the resulting FSMC model possesses memoryless observation property (MOP).

Fig. 4 shows the capacity estimates of the FF channel at the normalized fading rate $f_D T = 0.10$. BPSK signaling with soft detection is used. The FF channel phase is partitioned into $L = 2$ equiprobable intervals and the FSMC memory order ranges from $M = 1$ to $M = 4$. The FF channel amplitude is not approximated with the FSMC model and is assumed to be a memoryless Rayleigh-distributed process that is unknown at the receiver side. According to [10], FSMC modeling of the FF channel amplitude has a negligible effect on the channel capacity in the constant amplitude BPSK scheme. The FSMC capacity has been computed using the simulation-based numerical technique [7]. The graphs in Fig. 4 are plotted versus the SNR per symbol $\gamma_s = \frac{\mathcal{E}_s}{N_0}$ (averaged over all FF channel amplitudes). The non-monotonic behavior of capacity estimates with decreasing the FSMC memory order is observed. For example, suppose that the original FF channel phase has indeed a memory degree of $M = 4$. When $\text{SNR} \lesssim 5$ dB, the true capacity is underestimated by the third- and second-order FSMC models. If the channel memory order is further reduced to one, capacity overestimation occurs.

Next, the number of FF channel phase partition intervals is increased from $L = 2$ to $L = 8$ at the normalized fading rates $f_D T = 0.0125$, $f_D T = 0.05$, and $f_D T = 0.10$. The results are shown in Fig. 5. Three main points are observed from Fig. 5. First, as the normalized fading rate increases, the capacity difference ΔC also increases. Second, as the fading rate increases, $\Delta C > 0$ is observed only for lower SNR ranges. Third, compared to the two-level phase partitioning of the FF channel in Fig. 4, the maximum capacity difference between the first- and second-order FSMC models increases from $\Delta C = 0.0139$ bits/ch use to $\Delta C = 0.0763$ bits/ch use at the normalized fading rate $f_D T = 0.10$.

V. CONCLUSIONS

In this paper, we studied the effect of memory order on the capacity of a practical class of FSMC models with memoryless observation property (FSMC-MOP). We showed that although reducing the memory order of an FSMC-MOP model results in an FSMC-MOP model with a less predictable state process, the reduced-order FSMC capacity is not necessarily lower than the original high-order FSMC capacity.

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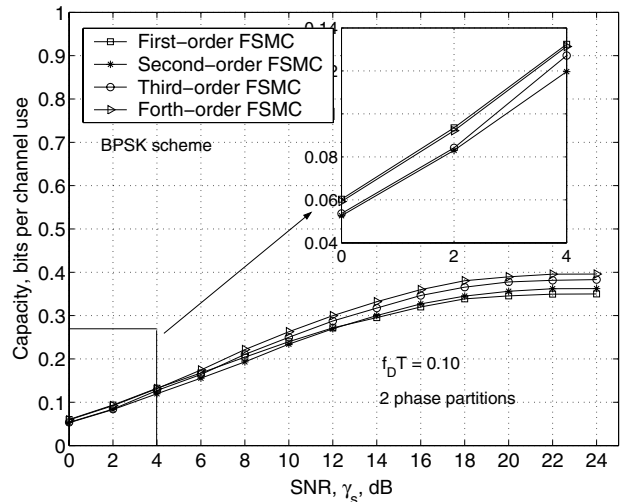


Fig. 4. Capacity estimates of the FF channel at the normalized fading rate $f_D T = 0.10$. Non-monotonic behavior of the capacity estimates with the assumed Markov memory order is observed in the figure.

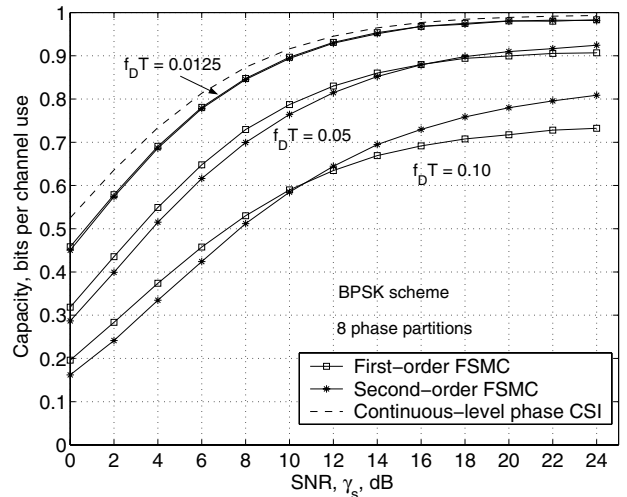


Fig. 5. Capacity estimates of the FF channel at various normalized fading rates. The FF channel phase is partitioned into $L = 8$ intervals.

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