AN INFORMATION-THEORETIC MEASURE FOR THE SELECTION OF EFFECTIVE MEMORY ORDER IN FLAT-FADING CHANNELS

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ABSTRACT

In this paper, we consider an information-theoretic measure for the selection of effective memory order in time-varying flat-fading (FF) channels in the Jakes-Clarke's model. The measure is based on how many past realisations of the fading channel actively contribute to an achievable information rate lower bound through the channel. The lower bound signifies the achievable information rate when the receiver is equipped with a sup-optimal mismatched decoding rule, which would have only been optimal for a finite-state Markov channel (FSMC) with a finite memory order. The sensitivity of the mismatched decoding rule to memory order is used to find the optimum channel memory order for decoding at the receiver side, with a negligible information rate loss. Our case study shows that the FF channel has an effective information-theoretic memory order of $M = 1$ for the normalised fading rates $f_D T \lesssim 0.01$, where f_D is the Doppler spread and T is symbol period. For faster fading rates, the use of first-order models at the receiver incurs a non-negligible information rate loss. The results are, generally, in accordance with our previous findings.

I. INTRODUCTION

I-A Background and Motivation

One of the earliest and most widely used models to describe flat-fading (FF) channels is the Jakes-Clarke's model [1], [2], which is relatively accurate for urban and suburban wireless communication channels [3], [4]. The Rayleigh FF channel observation equation in the Jakes-Clarke's model is characterised by a multiplicative, zero-mean, and Gaussian fading gain. Despite the simplicity of channel observation equation, there is still a number of open issues related to this model, especially when the FF channel is varying considerably over consecutive observation intervals.

It is known that, in principle, the FF channel in the Jakes-Clarke's model possesses an infinite memory order [5]. This means that the current realisation of FF channel gain depends on its infinite past realisations. However, for many practical applications, ranging from FF channel simulation to communication receiver design, assuming an infinite FF channel memory order is computationally prohibitive. For example, finite-state Markov channels (FSMC) [6] are widely used to model FF channels and to evaluate communication system performance measures, such as data packet error distribution [7], [8] or data block error rate [9] in wireless data protocols. Moreover, FSMC models have been proposed in [10]–[13] to be implemented at the receiver side for tracking fading channels, where the first-order FSMC modelling of fading channels is the most widely used in the literature [10]–[12], due to its low computational complexity and despite its potential inaccuracy in describing the original FF channel in the Jakes-Clarke's model [9].

Several authors have previously investigated the accuracy of finite memory order assumption for representing FF channels. The decision on *effective* channel memory order depends on the application and the measure of accuracy and as a result, there is not a unique consensus about the effective channel memory order in various fading regimes. By calculating the normalised mutual information between three successive fading channel amplitudes, [14] showed that the first-order FSMC model is accurate for normalised fading rates of $f_D T \lesssim 0.004$, where f_D is Doppler frequency spread and T is symbol period. However, [9] pointed out that the criterion in [14] can only predict whether a second-order FSMC model is marginally more accurate than a first-order FSMC model. [7], [15] proposed the context tree pruning method to find the best-fitting Markov model for the quantised fading channel amplitude with a desired fidelity criterion. The general conclusion in [7], [15] is that the first-order FSMC model can be accurate for the normalised fading rates $f_D T \lesssim 0.01$. For higher fading rates $0.01 \lesssim f_D T \lesssim 0.4$, a second- to fifth-order FSMC model is more appropriate. The above papers focus on applications where the FSMC is replacing the Jakes-Clarke's model for system performance evaluation. For FSMC models that are used at the receiver side in FF channels, [13] compared the BER performance of receivers that are based on the first- and second-order FSMC modelling of fading channels. Interestingly, with the same receiver computational complexity (i.e. an equal number of states in the first- and second-order FSMC models), the first-order FSMC modelling of the fading channel amplitude at the receiver outperforms the second-order FSMC modelling.

I-B Approach and Contributions

In this paper, we are interested to find how the FF channel memory order contributes to the channel capacity in time-varying fading regimes. For communication receivers that assume a finite channel memory order, an information-theoretic measure, such as the achievable information rate through the channel (with a finite memory order assumption at the receiver) seems more appropriate. However, the capacity of time-varying and continuous-valued FF channels (with either finite or infinite memory order and with no channel state information (CSI) at the receiver side) is still an open research problem [10]. In previous work [16]–[18], we modelled FF channels using FSMC models and then calculated the resulting FSMC capacities. However, the effect of increasing FSMC memory order on its capacity is not monotonically increasing, which is further explained in [17], [18]. The FSMC capacity non-monotonic behaviour with memory order hinders obtaining universal results about the optimum FF memory order at the receiver side with a negligible information rate loss across all SNR values.

In this paper, we use an alternative information-theoretic measure to find the effective FF channel memory order for the design of communication receivers. The new measure is based on the information rate lower bound for general (complicated) channels using auxiliary FSMC models, which was derived in [19]. By computing the lower bound using auxiliary FSMC models of increasing memory order, we can determine the optimum FF channel memory order with a negligible achievable information rate penalty. The lower bound also signifies the achievable information rate when the receiver is equipped with a sup-optimal mismatched decoding rule [20], which would have only been optimal for a FSMC with a finite memory order. The sensitivity of the mismatched decoding rule to memory order is used to find the optimum channel memory order for decoding at the receiver side, with a negligible information rate loss. The main contributions of this paper are summarised as follows:

1) For the cases that are studied here, we compare the information rate lower bound with the information rate upper bound assuming perfect CSI and also with the results in [17]. We can comfortably conclude that the FF channel has an effective information-theoretic memory order $M = 1$ for $f_D T \lesssim 0.01$.
2) For the cases that are studied here and for $0.01 \lesssim f_D T \lesssim 0.1$, the lower bound increases noticeably using the second or the third-order auxiliary FSMC models. For these fading rates, there is a non-negligible information rate penalty in assuming a first-order time evolution for the FF channel at the receiver side for mismatched decoding.

II. SYSTEM MODEL

We consider the low-pass model for the communication system in FF channels. The channel input symbol is denoted by x_k. The received signal y_k is defined in the observation equation as [3, p. 815]

$$y_k = c_k x_k + n_k = a_k e^{j\theta_k} x_k + n_k, \tag{1}$$

where n_k is the additive white Gaussian noise (AWGN) with variance $N_0/2$ per dimension and $c_k = a_k e^{j\theta_k}$ is the zero-mean and Gaussian-distributed FF channel gain, with FF channel amplitude a_k and FF channel phase θ_k and a normalised variance of $E\{|C_k|^2\} = 1$. Therefore, the average signal-to-noise ratio (SNR) is given by $\gamma_s = \mathscr{E}_s/N_0$, where $\mathscr{E}_s$ is the average transmitted symbol energy.

Usually in urban and suburban mobile communication systems there is no line of sight. In this case, the FF channel amplitude a_k is Rayleigh-distributed and the FF channel phase θ_k is uniformly-distributed between 0 and 2π as

$$f(a_k, \theta_k) = \frac{1}{2\pi} \frac{a_k}{\sigma^2} \exp\left(-\frac{a_k^2}{2\sigma^2}\right), \tag{2}$$

where, in fact, a_k and θ_k are statically independent. In the Jakes-Clarke's model, the normalised autocorrelation function (ACF) of in-phase and quadrature channel gain components $c_{I,k}$ and $c_{Q,k}$ is defined as [3, p. 809]

$$\begin{aligned} \rho_m &= E\{C_{I,k}, C_{I,k-m}\} = E\{C_{Q,k}, C_{Q,k-m}\} \\ &= 0.5 J_0(2\pi m f_D T), \end{aligned} \tag{3}$$

where J_0 is the zero-order Bessel function of the first kind. Also, f_D is the maximum Doppler frequency shift or Doppler spread and T is the transmitted symbol duration. Moreover, the in-phase and quadrature components of FF channel gain are independent from each other. That is, $E\{C_{I,k}, C_{Q,k-m}\} = 0$ for all k and m. In the auto-regressive (AR) modelling of the FF process, the state space equation is [21]

$$c_k = -\sum_{i=1}^{M} \alpha_i c_{k-i} + w_k, \tag{4}$$

where w_k is a complex white Gaussian process. The AR parameters α_i are calculated using the well-known Yule-Walker equations and ACF coefficients in (3) [21].

From (3), Doppler power spectral density (PSD) of the FF process in the Jakes-Clarke's model is [3, p. 809]

$$S(f) = \frac{1}{\pi f_D T \sqrt{1 - \left(\frac{f}{f_D T}\right)^2}}. \tag{5}$$

The PSD in (5) is a non-rational function [5]. This means that an accurate AR representation of the FF process requires an infinite (or at least a very large) memory order M in (4) [5]. On the other hand, it is always desirable to model the evolution of FF process by using an AR process with a finite and usually small memory order M. From a practical viewpoint, it is already known that despite the theoretical infinite memory order of FF channels, only a finite number of past channel realisations *actively contribute* to the current channel realisation. The practical choice of the FF channel gain memory order depends on the application and the criterion. The main application of this paper is the design of communication receivers that operate in FF channels modelled in (1)-(5), but due to computational complexities, the receiver performs the decoding according to a simpler, but sub-optimal rule by assuming a finite memory order evolution for the channel. For this purpose, we first briefly review how the original FF channel model in (1) with a complicated, infinite-valued, and infinite memory order state space in (4) is mapped into a FSMC with a finite memory order. This FSMC model may be later used for channel estimation and data decoding at the receiver side [10]–[13].

III. FSMC MODELLING OF FF CHANNELS

In this section, mapping of time-varying FF channels into M-th order FSMC models is discussed. There are three main steps that need to be taken in order to obtain an FSMC model for FF channels. This is summarised as follows:

- The FF channel gain (either amplitude, phase, or both) is partitioned into a finite number of non-overlapping intervals. These intervals form the FSMC substates in the M-th order FSMC model.
- Once the FSMC states are defined, state transition probabilities are derived. The $(M+1)$-th order statistics of the desired FF channel gain are required to obtain the FSMC state transition probabilities.

- The channel observation law in the FSMC modelling of FF channels is defined. Definition and calculation of the channel observation law depends on the type of modulation and data detection.

The FSMC substates are denoted by r and are obtained by partitioning the FF channel gain c into a number of non-overlapping intervals. In joint FF channel partitioning, FF channel amplitude a and FF channel phase θ in (1) are divided into L_a and L_θ partition intervals, respectively. The r-th joint partition interval is defined as

$$\begin{aligned} v_{r_a} &\triangleq \{a : a_{r_a} \leqslant a < a_{r_a+1}\}, \quad 0 \leqslant r_a < L_a, \\ v_{r_\theta} &\triangleq \{\theta : \theta_{r_\theta} \leqslant \theta < \theta_{r_\theta+1}\}, \quad 0 \leqslant r_\theta < L_\theta, \\ v_r &\triangleq \{v_{r_a}, v_{r_\theta}\}, \quad 0 \leqslant r < L, \end{aligned} \tag{6}$$

where the total number of partitions is $L = L_a L_\theta$. Joint partitions are numbered as $R = r \triangleq r_a L_\theta + r_\theta$, where r_a is the amplitude partition index and r_θ is the phase partition index. At time index k, the FSMC state s_k composes of the current and $M-1$ previous FF channel partition indices as

$$s_k \triangleq (r_k, \cdots, r_{k-M+1}) \Leftrightarrow \begin{cases} c_k & \in \quad v_{r_k} \\ & \vdots \\ c_{k-M+1} & \in \quad v_{r_{k-M+1}} \end{cases} \tag{7}$$

The stationary state probability is derived by integrating the M-dimensional probability density function (pdf) of the FF channel gain over the appropriate channel partition interval.

The state transition probability in the FSMC model is

$$P_{s's} = \Pr(S_k = s | S_{k-1} = s') = \frac{\Pr(S_k = s, S_{k-1} = s')}{\Pr(S_{k-1} = s')}. \tag{8}$$

According to the definition of the FSMC states, the numerator in (8) involves integration of the $(M+1)$-dimensional FF channel gain pdf over the appropriate FF channel gain partition regions.

The channel observation law in the FSMC modelling of FF channels directly depends on the type of signal modulation at the transmitter and signal detection at the receiver. In this paper, we consider soft detection of phase shift keying (PSK) as a widely-used and practical scheme, where the receiver has no *a priori* knowledge of the FF channel amplitude or phase.

In the U-ary PSK scheme [3, pp. 171-173], the modulated symbol is $x_k \triangleq \sqrt{\mathscr{E}_s} e^{j\phi_k} = \sqrt{\mathscr{E}_s} e^{j\frac{2\pi l}{U}}$ for the information symbols $l \in \{0, 1, ..., U-1\}$. ϕ_k is the symbol phase at time index k. The receiver may model the FF channel phase and amplitude as an M-th order FSMC model, and then estimate the channel with the use of the resulting FSMC model. Referring to (1), the pdf of the complex-valued received signal y_k given the channel input x_k, a_k, and θ_k is denoted by $f(y_k | x_k, a_k, \theta_k)$ and is basically the pdf of a complex-valued and white Gaussian variable with the mean $a_k e^{j\theta_k} x_k$ and the variance $N_0/2$. The channel observation law in the state s is written as

$$\begin{aligned} q(y_k | x_k, S_k = s) &= f(y_k | x_k, \{a_k, \theta_k\} \in v_{r_k}) \\ &= \frac{\int_{v_{r_k}} f(y_k | x_k, a_k, \theta_k) f(a_k, \theta_k) da_k d\theta_k}{\int_{v_{r_k}} f(a_k, \theta_k) da_k d\theta_k}, \end{aligned} \tag{9}$$

where $f(a_k, \theta_k)$ was defined in (2). It is noted that the dependence of the FSMC observation law in (9) only on the latest realisation of the channel is a natural consequence of the channel being *flat* and the noise process n_k being white in (1). Nevertheless, the FSMC state transition probabilities in (7)-(8) take the Markov memory order into account. It is important to note that although FSMC states with the same current substate r_k share the same channel observation law, combining those states *does not* result in a reduced-state FSMC model with the *same* capacity as the original. Because, ignoring the dependence of states on previous substates results in a less correlated state process with less structure and higher entropy rate. For more details, see [17].

IV. A LOWER BOUND ON THE INFORMATION RATE OF FF CHANNELS USING AUXILIARY FSMC MODELS

Once an FSMC model is obtained for the original FF channel, there are two possible options to obtain an information-theoretic criterion for the applicability and accuracy of the resulting FSMC model. The first option is to directly calculate the information capacity or the information rate of the resulting FSMC model. This approach was previously taken in [16], [17]. More specifically, it was shown in [17] that increasing the memory order in an FSMC does not necessarily have a monotonic effect on its capacity, where a low-order FSMC may have a higher capacity than a high-order FSMC model at the same channel observation conditions, especially in the low SNR regime. This is despite the fact the underlying state process in the low-order FSMC model is less predictable (has a higher entropy) than the state process in high-order FSMC model. This phenomenon was further explained in [17], [18]. This non-monotonic behaviour hinders drawing universal results about the effective channel memory order that actively contributes to the original FF channel capacity.

In this paper, we propose to use an alternative information rate measure that is based on the information rate lower bound derived in [19]. We start by briefly reviewing the lower bound that was originally derived for discrete memoryless channels (DMC). However, in the context of PSK signalling, discussed in the previous section, we will compute the bound in its continuous channel observation format.

Let X and Y be two discrete random variables with joint probability mass function $p(x,y)$, where X represents the source and $p(y|x)$ is the channel law of a DMC. Let $q(y|x)$ be the law of an *arbitrary auxiliary* channel with the same input and output alphabets as the original channel. We will imagine that the auxiliary channel is connected to the same source X; its output distribution is then

$$q_p(y) = \sum_x q(x) q(y|x). \tag{10}$$

The lower bound on the mutual information rate $I(X;Y)$ of the original channel (whose rate is difficult or impossible to compute) is then given by

$$\begin{aligned} I(X;Y) &\geq \sum_{x,y} p(x,y) \log \frac{q(y|x)}{q_p(y)} \\ &= E_{p(.,.)} \left[\log q(Y|X) - \log q_p(Y) \right]. \end{aligned} \tag{11}$$

For time-varying channels with memory the bound becomes

$$I(X;Y) \geq \lim_{n \to \infty} \frac{1}{n} E_{p(.,.)} \left[\log q(Y_1^n | X_1^n) - \log q_p(Y_1^n) \right]. \tag{12}$$

By using the FSMC models of FF channels, described in the previous section, as the auxiliary channel, this lower bound can be computed numerically using the simulation-based techniques in [19]. We have adapted the bound for the complex-valued and continuous-level channel output y in (1), which is much more computationally demanding. Two main features of (12) are that 1) the averaging is over all possible channel input and output realisations of the original (complicated) FF channel and 2) it guarantees a lower bound on the original FF channel information rate. This measure also becomes attractive from a receiver-design point of view, where the actual signal goes through the FF channel with the channel law $f(y|x)$, whereas the receiver is equipped with an FSMC-based mismatched decoding metric [20] such as $q(y|x)$ given in (9) (by further averaging over all FSMC states). The achievability of this lower bound with independent codewords and product distribution for codewords is pointed out in [20] and references therein.

V. Application of the Lower Bound for the Selection of Effective FF Channel Memory Order

The lower bound in (11) becomes tight if $p(x)q(y|x) = p(x,y)$ for all x and y, *i.e.* when the auxiliary channel has the same channel law as the original channel. In other words, the lower bound is a measure of similarity between the two channels. In this paper, we use this lower bound to measure the accuracy of finite M-th order FSMC models to be used at the receiver side for mismatched decoding. Due to the computational complexities, the assumed channel memory order M must be kept to a minimum. For instance, first-order FSMC models are most widely used at the receiver, due to their linear increase of computational complexity with increasing FF channel gain partitions or substates. In particular, we would like to know what is the maximum normalised fading rate, for which second or higher-order Markov modelling of the fading channel gives a marginal information rate improvement over the first-order Markov modelling.

Fig. 1 shows information rate lower bounds for the FF channel using auxiliary FSMC models at the relatively slow fading rate of $f_D T = 0.0125$ for BPSK signalling. Referring to (6), the FF channel gain is partitioned into $L_a = 1$ and $L_\theta = 2,4,8$ intervals. The memory order in the auxiliary FSMC ranges from $M = 1$ to $M = 3$. Other details are given in the figure caption. It is concluded that the first-order Markov modelling of FF channels for this fading rate is just enough and does not incur information rate loss. For comparison, the information rates using the direct calculation of the resulting FSMC capacities [17] are also shown. Unlike (12), where the averaging is over possible original FF channel input/outputs with pdf $p(x,y)$, in the direct FSMC information rate calculation the averaging is over channel input/outputs according to the FSMC pdf $q(x,y)$. The information rates are close to the lower bound and also do not exhibit sensitivity to the FSMC memory order, which is in accordance with the new measure. This confirms that even replacing the FF channel with a first-order FSMC model generates accurate estimates of the channel capacity. However, it is noted that in the direct capacity calculation method, the second-order FSMC model has a slightly lower capacity than the first-order FSMC model. This phenomenon is more pronounced for faster FF channels and is further explained in [17].

Fig. 2 shows information rate lower bounds for the FF channel using auxiliary FSMC models at a faster fading rate of $f_D T = 0.10$ for BPSK signalling. Other channel details and the scenarios for the computation of information rates are identical to Fig. 1. A clear distinction between the graphs in Fig. 1 and 2 is that the information rates are sensitive to the assumed auxiliary FSMC memory order. For example, the second-order FSMC-based mismatched decoding rule with 8 channel phase partitions can achieve the same information rate of 0.5 bits / channel use at about 2.7 dB lower SNR than the first-order FSMC-based mismatched decoding rule. The price is, of course, the computational complexity that increases by using 64 FSMC states instead of 8. Due to computational limitations, lower bounds with higher FSMC memory orders could not be computed. As a result, one cannot draw conclusive results on how the FF channel capacity saturates with memory order. Nonetheless, it can be comfortably deduced that the effective channel memory order is $M > 1$. The direct calculation of FSMC information rate also produces results that are sensitive to the channel memory order, but in a non-monotonic manner. The results are not shown for the clarity of figure. In any case, the information rate sensitivity to memory order can be used as a criterion to understand the tradeoffs in the design of communication receivers in time-varying FF channels.

VI. Conclusions

In this paper, we studied the effective FF channel memory order that actively contributes to the achievable information rate through the channel. For this purpose, we used an information rate lower bound which uses auxiliary FSMC models with various degrees of memory order. This criterion may be used for the design of FSMC-based communication receivers with the optimum computational complexity and a negligible information rate loss due to mismatched decoding. It was concluded that the first-order Markov modelling of the FF channel is accurate and efficient for the normalised fading rates of $f_D T \lesssim 0.01$. For faster fading rates $0.01 \lesssim f_D T \lesssim 0.1$, first-order Markov modelling of the FF channel at the receiver side incurs a non-negligible information rate loss.

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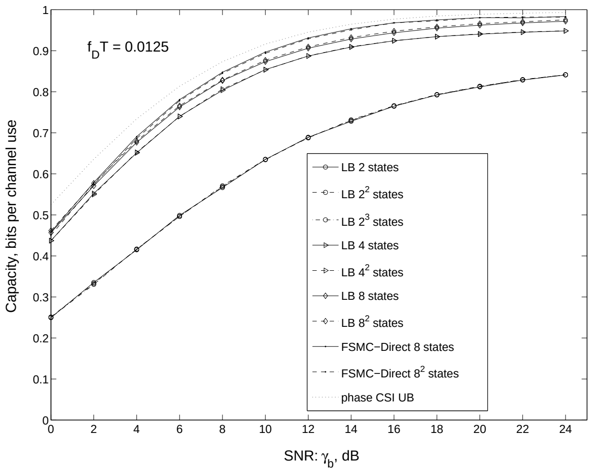

Fig. 1. Information rate lower bounds for the FF channel using auxiliary FSMC models. The number of FF channel phase substates in the FSMC model is 2 (○), 4 (▷), and 8 (◇). The FSMC memory order is $M = 1$ (solid line), $M = 2$ (dashed line), and $M = 3$ (dashed-dot line). The increase of information rate lower bounds with increasing M in negligible. The capacity upper bound with perfect phase CSI is also shown. There is not a considerable gap between the lower bounds and the upper bound. Therefore, it is concluded that the first-order Markov modelling of FF channels for this fading rate is just enough. For comparison, the information rates using the direct calculation of the resulting FSMC capacities [17] are shown. The information rates are close to the lower bounds and also do not exhibit sensitivity to the FSMC memory order, which is in accordance with the new measure.

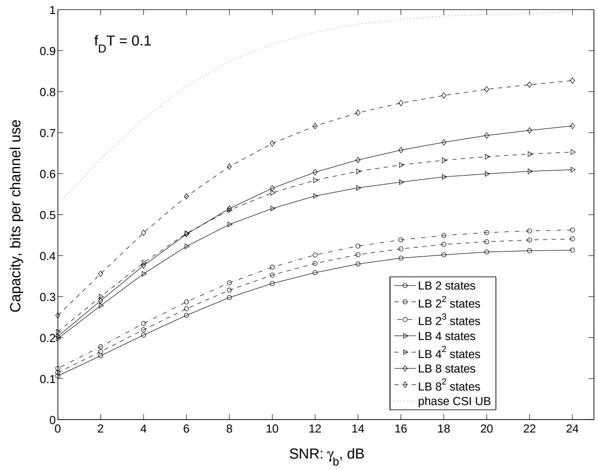

Fig. 2. Information rate lower bounds for the FF channel using auxiliary FSMC models. The number of FF channel phase substates in the FSMC model is 2 (○), 4 (▷), and 8 (◇). The FSMC memory order is $M = 1$ (solid line), $M = 2$ (dashed line), and $M = 3$ (dashed-dot line). The information rate lower bound shows noticeable sensitivity to the memory order M in the mismatched decoding rule. It can be concluded that for $f_D T = 0.1$, the effective information-theoretic channel memory order is at least $M = 2$.

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