

Capacity Analysis of Time-Varying Flat-Fading Channels Using Particle Methods

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Abstract: *We consider the problem of computing the information rate of a first-order approximation of Clarke's time-varying flat-fading channel model, and apply the recently-developed particle-filter method for estimating the information rate of a continuous-valued channel with memory. We compare the results with various bounds, both classic and recent, and we find that the information rates obtained using the particle method fall between recently-established tight bounds. We also investigate the effects of the numbers of particles and of simulated time-steps on the estimated information rate. Such results are useful in providing an upper bound on the data rates achievable in a wireless communication system.*

Keywords: fading channel, particle method, information rate, channel capacity, wireless communications.

I. INTRODUCTION

A. Background

Computation of the information rate of a channel with memory, in general, requires time that is exponential in the channel input/output window-length; for accurate results, the window-length must be large. In particular, for a time-varying flat-fading channel with memory that behaves in accordance with Clarke's model [1] and for which channel-state information (CSI) is not available at the receiver, computation of the information rate is a long-standing open problem [2].

A tractable method now exists for computing the information rate of a channel with memory provided that it has a finite state-space [3], [4], [5]. This method cannot be applied, however, to a channel with an infinite number of states, such as a continuous-valued channel. Very recently a particle-filter framework has been developed to compute the approximate information rate of a continuous channel with memory [6]; this particle-filter method has not yet been applied to fading channels.

From a practical point of view, modelling of fading channels and computation of their information rates shall be significant in the design of future wireless communication systems. By taking time-variations and memory of a fading channel into account at the receiver, novel decoding techniques could be devised that avoid capacity loss due to interleaving [7], [8].

Moreover, having an estimate of the information rate of a fading channel would provide an upper bound on the data rates achievable in a future wireless communication system.

Since direct computation of information rates for time-varying fading channels with infinite state-space had not previously been possible, a different approach was taken in [9], in which fading channels were modelled as finite-state Markov channels (FSMCs); their information rates were then computed using the available techniques proposed in [3].

Upper and lower bounds on the information rate were provided in [10] for arbitrary channels with memory, using auxiliary FSMC models. This work was later extended in [11], parameters of the auxiliary model being optimally chosen so as to make the bounds as tight as possible.

B. Contribution

In the current paper, we investigate the applicability of the particle-filter method to time-varying fading channels. In particular, we study the convergence of the estimated information rate as the number of particles and the number of iterations are increased independently; we also compare particle-filter estimates with an auxiliary-FSMC bound and with a bound obtained by assuming perfect CSI at the receiver. We restrict our investigation to the flat-fading channel modelled as a Gauss-Markov process; the Gauss-Markov model for flat-fading channels is widely accepted and studied (see [12] and [13] for examples).

The remainder of the paper proceeds as follows. In section II we review existing models for the fading channel, from the perspective of computing the channel's information rate. In section III we evaluate existing methods for the computation of a fading channel's information rate, including exact and approximate techniques for finite-state channel models, and methods for computing bounds on the information rate. In section IV we review the particle-filter method and apply it to Clarke's fading-channel model of finite memory-order. In section V we provide numerical analysis of the results and compare them with those obtained using other techniques. We conclude in section VI.

II. COMMUNICATION MODELS OF FADING CHANNELS

A. Clarke's flat-fading communication model

We outline a communication model for time-varying flat-fading channels. The model assumes that time is discretised into instants k separated by increments of T_s , the symbol period, so that at each discrete instant a new symbol is received.

The observation equation for the discrete-time channel consists (see p. 815 of [14]) of the transmitted signal x_k at the k th time-step undergoing a time-variant multiplicative *channel gain* h_k observed under additive white Gaussian thermal noise z_k , i.e.,

$$y_k = h_k x_k + z_k, \quad (1)$$

where y_k is the received signal. Because y_k depends on no input signals other than the current input x_k there are no delays or echoes, meaning that the channel, as required, exhibits frequency-nonselctive (flat) fading. In fact, given the current channel gain, the current output y_k is independent of all past and future inputs and outputs.

In Clarke's model there is assumed to be no line of sight between the transmitter and the receiver, and the transmission medium is assumed to exhibit uniform scattering. This implies that the channel gain h_k is a wide-sense-stationary zero-mean complex-valued circularly symmetric Gaussian random process, with real and imaginary components that are statistically independent of each other. Hence, if the flat-fading channel gain at time k is expressed using polar coordinates as $h_k = \alpha_k e^{j\theta_k}$, then the channel amplitude α_k is found to be Rayleigh-distributed, and the channel phase θ_k uniformly distributed over $[0, 2\pi)$.

An argument invoking the central limit theorem implies that the thermal noise is a Gaussian-distributed complex random variable $z_k \sim N_{\mathbb{C}}(0, N_0)$ for some variance N_0 , with the real and imaginary components statistically independent of one another and each having variance $\frac{N_0}{2}$.

Clarke's model assumes that the angle of incidence of incoming multipath components is uniformly distributed between 0 and 2π . The Doppler power spectrum is then found (upon normalising the power) to be the U-shaped function

$$S_C(\lambda) = \begin{cases} \frac{1}{\pi f_D} \frac{1}{\sqrt{1-(\lambda/f_D)^2}}, & |\lambda| \leq f_D; \\ 0, & |\lambda| > f_D, \end{cases}$$

where f_D is the maximum Doppler frequency-shift.

The autocovariance function $\phi_C(\Delta t)$ is obtained by taking the inverse Fourier transform of $S_C(\lambda)$, which leads to

$$\phi_C(\Delta t) = J_0(2\pi f_D \Delta t),$$

where $J_0(x)$ is the zeroth-order Bessel function of the first kind. (The normalisation just noted has the effect of setting the variance of the fading-channel gain to 1.)

B. Autoregressive models

It is convenient to model the fading channel gain h as an autoregressive process [15]; such a model has useful mathematical properties, and is easy to simulate because its memory-order is finite.

A complex-valued autoregressive process $\{u_k\}$ of finite order p satisfies

$$u_k = \sum_{m=1}^p a_m u_{k-m} + w_k,$$

where the a_m are the autoregressive filter coefficients and w_k is complex-valued white Gaussian process-noise with variance σ_w^2 . This process has a power density spectrum of the rational form [15]

$$S_{uu}(\lambda) = \frac{\sigma_w^2}{|1 - \sum_{m=1}^p a_m \exp(-j2\pi m\lambda)|^2}. \quad (2)$$

The power density spectrum $S_C(\lambda)$ of Clarke's channel is an irrational function of λ , and hence [16] it cannot correspond to a time-variant transfer function h that is an autoregressive process of finite order p . Therefore an infinite channel memory-order is implied by Clarke's model. Consequently the fading is non-Markovian, although it can potentially be approximated [15] by an autoregressive process of large but finite order p .

Given the desired order p for an autoregressive model, the filter coefficients a_m can be found using the Yule-Walker equations [15]

$$\phi_{uu}(d) = \begin{cases} \sum_{m=1}^p a_m \phi_{uu}(d-m), & d \geq 1, \\ \sum_{m=1}^p a_m \phi_{uu}(-m) + \sigma_w^2, & d = 0, \end{cases}$$

where $\phi_{uu}(d)$ is the d th autocorrelation coefficient $\frac{1}{2}E(u_k^* u_{k-d})$.

Let us therefore assume that the channel gain h_k varies with time according to the autoregressive model of memory order M ,

$$h_k = \sum_{m=1}^M a_m h_{k-m} + w_k, \quad (3)$$

where the filter coefficients a_m and the process-noise variance σ_w^2 are found by applying the Yule-Walker equations with $\phi_{hh}(d) = \frac{1}{2}E(h_k^* h_{k-d}) = \phi_C(dT_s) = J_0(2\pi f_D dT_s)$.

Because in the model (3) the current value of the channel gain h_k depends only on the previous M values, the sequence of gains is now approximated by a Markovian process of order M . In particular, the widely-used *Gauss-Markov* channel model has memory of order 1, and the channel gain satisfies the equation

$$h_k = ah_{k-1} + w_k, \quad (4)$$

where the Yule-Walker equations imply that $a = \phi_{hh}(1) = J_0(2\pi f_D T_s)$ and $\sigma_w^2 = 1 - a^2$. The gain is distributed as $h_k \sim N_{\mathbb{C}}(0, \phi_{hh}(0)) = N_{\mathbb{C}}(0, 1)$.

III. EXISTING METHODS FOR CAPACITY ANALYSIS OF CHANNELS WITH MEMORY

For a given FSMC model, there are three main methods for computing the capacity. Where the channel is modelled as something other than an FSMC, there exist

various general approaches to obtaining bounds on the capacity; one recent approach is to use the notion of an *auxiliary FSMC* to compute bounds. This is discussed briefly in the following subsections.

A. Methods applicable to a general channel

1) *Perfect-CSI upper bounds*: A well-known upper bound on the information rate of a channel can be obtained by assuming that at all times, the receiver has perfect CSI [17].

If binary phase-shift-keying (BPSK) signalling is used, then the channel observation equation may be written

$$y = hx + z = \alpha e^{j\theta} x + z, \quad (5)$$

where the channel gain's amplitude α has the Rayleigh distribution with $\sigma^2 = 0.5$ (half the total variance of the channel gain), the channel gain's phase θ is uniformly distributed between 0 and 2π , $x \in \{-\sqrt{\mathcal{E}_x}, \sqrt{\mathcal{E}_x}\}$, and $z \sim N_C(0, N_0)$. Multiplying both sides of (5) by $e^{-j\theta}$, and noting that αx is real, we have $\text{Re}(ye^{-j\theta}) = \alpha x + \text{Re}(ze^{-j\theta})$. As the noise z has a circularly-symmetric Gaussian distribution, the statistics of z are invariant under rotation. Hence, defining $y' = \text{Re}(ye^{-j\theta})$ and $z' = \text{Re}(ze^{-j\theta})$, the observation equation is

$$y' = \alpha x + z'$$

where $z' \sim N(0, \frac{N_0}{2})$. Now the information rate with perfect CSI is

$$\bar{I}_{CSI} = h(y'|\alpha) - h(y'|x, \alpha), \quad (6)$$

with

$$h(y'|x, \alpha) = - \sum_x p(x) \int_0^\infty f(\alpha) \int_{-\infty}^\infty f(y'|x, \alpha) \times \log f(y'|x, \alpha) dy' d\alpha$$

where $f(\alpha) = \frac{\alpha}{\sigma^2} \exp\left(-\frac{\alpha^2}{2\sigma^2}\right)$ and $Y|x, \alpha \sim N(\alpha x, \frac{N_0}{2})$, so that $f(y'|x, \alpha) = \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(y' - \alpha x)^2}{N_0}\right)$. It is then found that

$$h(y'|x, \alpha) = \frac{1}{2} \log(\pi e N_0).$$

On the other hand $h(y'|\alpha) = - \int_0^\infty f(\alpha) \int_{-\infty}^\infty f(y'|\alpha) \log f(y'|\alpha) dy' d\alpha$, where $f(y'|\alpha) = \sum_x p(x) f(y'|x, \alpha)$. Assuming X to be uniformly distributed, we have $p(x) = \frac{1}{2}$ and

$$f(y'|\alpha) = \frac{1}{\sqrt{\pi N_0}} \exp\left(\frac{-y'^2 - \alpha^2 \mathcal{E}_x}{N_0}\right) \times \cosh\left(\frac{2\alpha y' \sqrt{\mathcal{E}_x}}{N_0}\right).$$

2) *Auxiliary-FSMC methods for computing bounds on the information rate*: In [10] it is shown that upper and lower bounds on the information rate of an arbitrary infinite-state (stationary and ergodic) channel with memory can be computed using the simulation method of subsection III-B.3 below. This is achieved by way of an auxiliary finite-state channel that approximates the given channel, in that simulated pairs

of input and output values from the actual channel are used in the computational steps of the simulation method applied to the trellis of the auxiliary channel. Computation of the upper bound requires an existing lower bound on the conditional entropy rate $h(Y|X)$ of the actual channel.

The work of [10] was later extended in [11]. For a given number of states in the auxiliary FSMC, other parameters of the model are optimised in an iterative expectation-maximisation (EM) manner so as to minimise the upper bound and maximise the lower bound.

B. Methods applicable to an FSMC

The three main methods for computing the capacity of an FSMC model are (i) recursive methods for calculating the exact capacity, (ii) computation of upper and lower bounds on the capacity using direct evaluation of the entropy functions, and (iii) stochastic-numerical simulation techniques for approximating the capacity.

The first two of these methods are computationally complex, while the simulation technique requires time that is merely linear in the number of simulated time-steps.

We use the notation $x^n \equiv x_1^n \equiv (x_1, x_2, \dots, x_n)$ to denote a sequence of n values of the channel input-process X , and we use y^n and s^n similarly with respect to the channel output-process Y and state-process S , respectively.

1) *Recursively computing the exact information rate*: These methods are practical only for a very small number of FSMC states [7], [8].

2) *Upper and lower bounds using properties of HMMs*: An upper bound and a lower bound can be derived using properties of hidden Markov models (HMMs) [18]. Computation of these bounds [19] requires evaluation of entropies of the form $h(Y_1^D)$ and $h(Y_1^D|S_0)$, where the number of time-steps is limited to D . If the channel output y can take Q possible discrete values, and there are N possible channel states s , then such evaluation requires NQ^D probability-mass points to be calculated: this is a prohibitively large number for realistically large values of D , Q , and N .

3) *Simulation methods for estimating the information rate*: Based on the BCJR algorithm [20], a method for computing the information rate of a general finite-state source/channel model was independently reported by [3], [4], and [5]. This assumes that the input-process X , output-process Y , and finite-valued state-process S satisfy, for $n > 0$, the factorisation

$$p(x^n, y^n, s_0^n) = p(s_0) \prod_{k=1}^n p(x_k, y_k, s_k | s_{k-1}) \quad (7)$$

and that $p(x_k, y_k, s_k | s_{k-1})$ is independent of k . Two "very long" sample sequences x^n and y^n are generated, and are used in conjunction with the asymptotic equipartition property (A.E.P.) [21] to obtain approximations for $h(Y)$ and for either $h(Y|X)$ or $h(X, Y)$. The approximate information rate

$$\hat{I}(X; Y) = \hat{h}(Y) - \hat{h}(Y|X) \quad (8)$$

can then be calculated.

In order to obtain an approximation for $h(Y)$, for example, we define the *state metric* $\mu_k(s_k) = p(s_k, y^k)$ for $k \geq 0$ and iteratively apply the forward sum-product recursion

$$\mu_k(s_k) = \sum_{x_k, s_{k-1}} \mu_{k-1}(s_{k-1}) p(x_k, y_k, s_k | s_{k-1})$$

of the BCJR algorithm to the factor graph ([22], [23]) corresponding to (7).

The state metric $\mu_k(s_k)$ rapidly approaches zero as k increases, so in practice it is normalised to $\mu'_k(s_k) = \lambda_k \mu_k(s_k)$ in such a way as to ensure that

$$\sum_{s_k} \mu'_k(s_k) = 1. \quad (9)$$

The normalised version of the forward recursion is then

$$\mu'_k(s_k) = \lambda_k \sum_{x_k, s_{k-1}} \mu'_{k-1}(s_{k-1}) p(x_k, y_k, s_k | s_{k-1}), \quad (10)$$

which implies that

$$\mu'_k(s_k) = p(s_k | y^k) \quad \text{and} \quad \lambda_k = \frac{1}{p(y_k | y^{k-1})}$$

for $k \in \mathbb{N}^+$. Hence by the A.E.P.,

$$\begin{aligned} h(Y) &\approx -\frac{1}{n} \log p(y^n) = -\frac{1}{n} \log \prod_{k=1}^n p(y_k | y^{k-1}) \\ &= -\frac{1}{n} \sum_{k=1}^n \log \frac{1}{\lambda_k} = \frac{1}{n} \sum_{k=1}^n \log \lambda_k. \end{aligned}$$

The quantity $h(X, Y)$ can be estimated similarly.

IV. THE PARTICLE METHOD ON A TIME-VARYING FADING CHANNEL

The particle-filter method [6] is a generalisation of the BCJR algorithm to the case in which the state-process S is continuous-valued. As before, a ‘‘very long’’ sample sequence y^n is generated. Each state metric μ'_k is represented by a list $\{\hat{s}_{k,l}\}_{l=1}^M$ of M samples, where M is a large number, and each probability $\mu'_{k-1}(s_{k-1}) p(x_k, s_k | s_{k-1})$ is represented by a list of M three-tuples $(\hat{s}_{k-1,l}, \hat{x}_{k,l}, \hat{s}_{k,l})$. We assume the continuous-valued analogue of (9), namely

$$\int_{s_k} \mu'_k(s_k) = 1,$$

and use this with the continuous-valued analogue

$$\mu'_k(s_k) = \lambda_k \int_{x_k, s_{k-1}} \mu'_{k-1}(s_{k-1}) p(x_k, y_k, s_k | s_{k-1}) \quad (11)$$

of the forward recursion (10) to deduce that

$$\begin{aligned} \lambda_k^{-1} &= \int_{s_k, x_k, s_{k-1}} \mu'_{k-1}(s_{k-1}) p(x_k, y_k, s_k | s_{k-1}) \\ &= \int_{s_k, x_k, s_{k-1}} p(s_{k-1} | y^{k-1}) p(x_k, s_k | s_{k-1}) \times \\ &\quad p(y_k | x_k, s_k, s_{k-1}) \\ &= \int_{s_k, x_k, s_{k-1}} p(s_{k-1}, x_k, s_k | y^{k-1}) \times \\ &\quad p(y_k | x_k, s_k, s_{k-1}) \\ &= E_{S_{k-1}, X_k, S_k | y^{k-1}} [p(y_k | X_k, S_k, S_{k-1})] \\ &\approx \frac{1}{M} \sum_{l=1}^M p_{Y_k | X_k, S_k, S_{k-1}}(y_k | \hat{x}_{k,l}, \hat{s}_{k,l}, \hat{s}_{k-1,l}). \end{aligned}$$

The algorithm is initialised by generating the samples $\{\hat{s}_{0,l}\}_{l=1}^M$ that represent μ'_0 . At each iteration of the forward recursion (11), each sample $\hat{s}_{k-1,l}$ is extended to a three-tuple $(\hat{s}_{k-1,l}, \hat{x}_{k,l}, \hat{s}_{k,l})$ by means of sampling from the distribution $X_k, S_k | S_{k-1}$, and the conditional probabilities $\{p(y_k | \hat{x}_{k,l}, \hat{s}_{k,l}, \hat{s}_{k-1,l})\}_{l=1}^M$ are calculated and used to compute the value of λ_k . The tuples $(\hat{s}_{k-1,l}, \hat{x}_{k,l}, \hat{s}_{k,l})$ are then resampled according to their likelihoods under the conditional probabilities; the values $\hat{s}_{k-1,l}$ and $\hat{x}_{k,l}$ are dropped from each tuple in order to obtain the next list $\{\hat{s}_{k,l}\}_{l=1}^M$ of samples.

In the case of the first-order model (4) of the flat-fading channel, we have $s_k = h_k$. The conditional probabilities are given by the observation equation (1).

If constant-power signalling is employed then $h(Y|X)$ can be evaluated analytically (see below), and the particle method is needed to compute $h(Y)$ only.

A. Analytical evaluation of $h(Y|X)$

Assuming constant-power signalling, a closed form for $h(Y|X)$ is obtained as follows [17].

The received signal at time k is given by $y_k = h_k x_k + z_k$, so a block of L transmitted symbols $\mathbf{x} = (x_1 \ x_2 \ \dots \ x_L)^t$ satisfies the equation

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{z}, \quad (12)$$

where $\mathbf{y} = [y_1 \ y_2 \ \dots \ y_L]^t$, $\mathbf{X} = \text{diag}(x_1, x_2, \dots, x_L)$, $\mathbf{h} = [h_1 \ h_2 \ \dots \ h_L]^t$, $\mathbf{z} = [z_1 \ z_2 \ \dots \ z_L]^t$, and \mathbf{z} is white noise having an L -dimensional complex Gaussian distribution with covariance matrix $N_0 \mathbf{I}$. The set of fading samples \mathbf{h} is complex Gaussian with zero mean and covariance matrix \mathbf{R}_h , and is independent of \mathbf{z} . Referring to (4), we see that \mathbf{R}_h is a symmetric Toeplitz matrix with (i, j) -th entry $a^{|i-j|}$.

Given \mathbf{X} , the received sequence \mathbf{y} has an L -dimensional complex Gaussian distribution with mean $\mathbf{0}$ and covariance matrix $E[\mathbf{y}\mathbf{y}^*] = \mathbf{X}\mathbf{R}_h\mathbf{X}^* + N_0\mathbf{I}$. Let Σ denote this covariance matrix. Then

$$\begin{aligned} h(\mathbf{y}|\mathbf{x}) &= -E_{\mathbf{x}} \int_{\mathbf{y}} \frac{\exp(-\mathbf{y}^* \Sigma^{-1} \mathbf{y})}{\pi^L \det \Sigma} \times \\ &\quad \log \left(\frac{\exp(-\mathbf{y}^* \Sigma^{-1} \mathbf{y})}{\pi^L \det \Sigma} \right) d\mathbf{y} \\ &= E_{\mathbf{x}} E_{\mathbf{y}|\mathbf{x}} [(\log e)(\mathbf{y}^* \Sigma^{-1} \mathbf{y}) + \log(\pi^L \det \Sigma)]. \end{aligned}$$

But for a constant square matrix \mathbf{A} , and a random vector \mathbf{w} with mean μ and covariance matrix \mathbf{D} , it

is well known that $E[\mathbf{w}^t \mathbf{A} \mathbf{w}] = \text{trace}(\mathbf{A} \mathbf{D}) + \mu^t \mathbf{A} \mu$. Therefore

$$\begin{aligned} h(\mathbf{y}|\mathbf{x}) &= E_{\mathbf{x}}[(\log e)(\text{trace}(\boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}) + 0) + \\ &\quad \log(\pi^L \det \boldsymbol{\Sigma})] \\ &= E_{\mathbf{x}} \log((\pi e)^L \det \boldsymbol{\Sigma}) = E_{\mathbf{x}} \log \det(\pi e \boldsymbol{\Sigma}) \\ &= L \log(\pi e N_0) + \\ &\quad E_{\mathbf{x}} \log \det \left(\mathbf{I} + \frac{1}{N_0} \mathbf{R}_h \mathbf{X}^* \mathbf{X} \right), \quad (13) \end{aligned}$$

by the identity $\det(\mathbf{I} + \mathbf{A} \mathbf{B}) = \det(\mathbf{I} + \mathbf{B} \mathbf{A})$. We have assumed that the input symbols are of constant power $|x_k|^2 = \mathcal{E}_x$, as in the case of M -ary phase-shift-keying (M -PSK). Hence, normalising (13) by the number of symbols, we obtain

$$\frac{1}{L} h(\mathbf{y}|\mathbf{x}) = \log(\pi e N_0) + P_{\Delta},$$

where

$$P_{\Delta} = \frac{1}{L} \log \det \left(\mathbf{I} + \frac{\mathcal{E}_x}{N_0} \mathbf{R}_h \right).$$

We wish to evaluate the limit of P_{Δ} as $L \rightarrow \infty$. Using Szegő's theorem [24] and referring to (2) for the Gauss-Markov channel model (4), we obtain

$$\begin{aligned} \lim_{L \rightarrow \infty} P_{\Delta} &= \frac{1}{2\pi} \int_0^{2\pi} \log \left(1 + \frac{\mathcal{E}_x}{N_0} S_{hh} \left(\frac{\omega}{2\pi} \right) \right) d\omega \\ &= \frac{1}{2\pi} \int_0^{2\pi} \log \left(1 + \frac{\mathcal{E}_x(1-a^2)}{N_0(1-2a \cos \omega + a^2)} \right) d\omega. \end{aligned}$$

This is evaluated using a method of numerical integration, and the per-symbol entropy of $Y|X$ is hence obtained.

V. NUMERICAL RESULTS

The convergence of the particle method's estimates as the number n of iterations increases is shown in Fig. 1. The estimates are generated for each of ten runs using $M = 500$ particles, a signal-to-noise ratio (SNR) of 7 dB, and the fading rate $f_D T_s = 0.1$. The results show that for $n \gtrsim 2^{15}$, the estimates over ten runs have a range of only $\pm 2.5\%$ of their average value (and have converged to about 0.68, which agrees with the estimate in Fig. 3 below).

Fig. 2 shows the convergence of the particle method's estimates as the number M of particles is increased. The estimates are generated for each of ten runs using $n = 2^{15}$ iterations, a fading rate of 0.1, and an SNR of 7 dB. Where the number of particles is low, all ten estimates are extremely inaccurate; however the variance of the estimates is similarly small regardless of the number of particles. It is seen that the mean estimate of $I(X;Y)$ over ten runs has converged by $M \gtrsim 2^9$ (to about 0.68, which agrees with the estimate in Fig. 3 below).

Fig. 3 plots the estimated information rate (8) given by the particle method, and the CSI upper bound (6) for BPSK signalling, as functions of SNR. The estimates are generated using $M = 512$ particles, $n = 10^5$ iterations, and the fading rate $f_D T_s = 0.1$.

Fig. 3 also provides a new lower bound on the information rate of the flat-fading channel, obtained

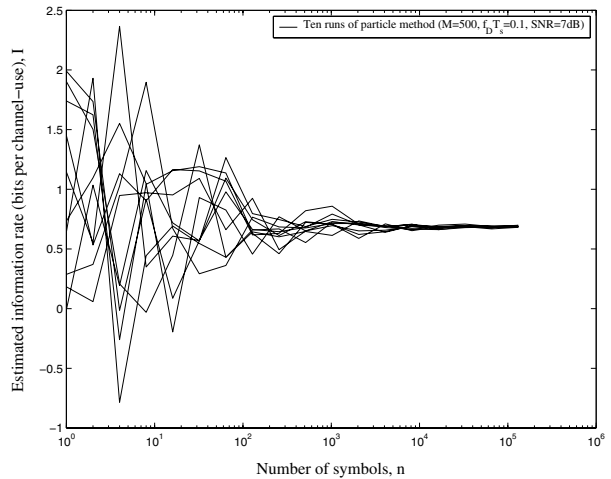


Fig. 1. Particle-method estimate of information rate versus the number of iterations n .

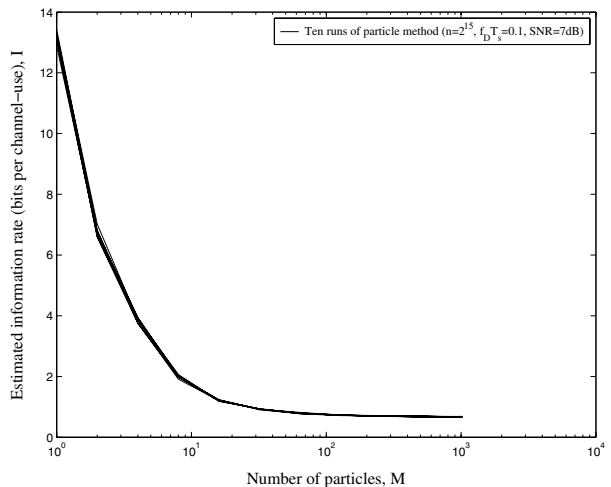


Fig. 2. Particle-method estimate of information rate versus the number of particles M .

by applying the technique recently proposed in [11]. For this purpose, we use an auxiliary FSMC model with 16 different states to represent the channel gain h_k , the channel phase θ_k being partitioned into eight levels and the channel amplitude α_k being partitioned into two levels (see [25] for more details of the mapping of the channel phase to an FSMC model). With this initial choice of the auxiliary FSMC model, we apply the optimisation technique for iteratively minimising the difference between the upper bound and the lower bound. The optimisation algorithm is run for 300 iterations. Subsequently, we use the optimised parameters of the auxiliary FSMC model (namely, its state-transition probabilities and channel observation law in each state) to compute the lower bound on the information rate. It should be noted that in order for the optimisation technique to work properly, the number of channel-output levels must be finite. This does not pose a problem, however, since quantising the channel output inherently provides a lower bound on the information rate (higher information rates are possible if the receiver works with purely soft or continuous-valued channel output). Therefore, we have a lower bound in

two senses, firstly by using an auxiliary FSMC model for the original flat-fading channel and secondly by quantising the output. We have used the following output quantisation thresholds per real and imaginary dimension:

$$y_q = [-\infty, -2.5\sigma_y, -1.5\sigma_y, -1.0\sigma_y, -0.5\sigma_y, 0.0, 0.5\sigma_y, 1.0\sigma_y, 1.5\sigma_y, 2.5\sigma_y, \infty].$$

Here σ_y^2 is the average output variance per dimension and is defined as $\sigma_y^2 = \mathcal{E}_x + N_0/2$.

It is seen that the lower bound is close to the particle-method estimates. this indicates the suitability of both methods for providing estimates of information rates of fading channels.

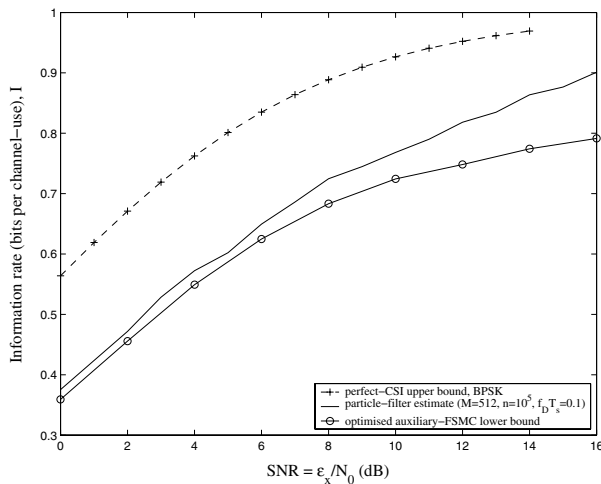


Fig. 3. Information rate (particle-method estimate and bounds) versus SNR.

VI. CONCLUSIONS

We have applied the particle-filter method to the flat-fading channel, with channel gain modelled as a first-order (Gauss-Markov) autoregressive process.

Experiments show that the particle method converges with an increasing number of iterations in such a way that its estimates are accurate for all n but are precise only for large n . On the other hand, the method gives estimates that are precise for all values of M , the number of particles, but are accurate only for large M .

We see that the particle method provides estimates that in every case lie between the theoretical upper bound for perfect CSI and the lower bound given by an optimised auxiliary FSMC model. However, the upper bound is very loose for this relatively fast-fading channel, the assumption of perfect CSI being inaccurate in this case. On the other hand, the lower bound and the particle method provide close estimates of the information rate; this is evidence for the suitability of both methods for providing estimates of information rates of fading channels.

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