

# A NEW POWER ALLOCATION SCHEME FOR AMPLIFY AND FORWARD INCREMENTAL RELAYING

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## ABSTRACT

In this paper, we present a novel power allocation scheme for amplify and forward incremental relaying (IR) systems with the aim to minimize the overall outage probability at the destination. In our problem formulation, we take into account the outage-dependent activation of the relay when writing the power constraint. We provide a simple closed-form solution to the IR power allocation problem that is based on high signal-to-noise (SNR) approximation of the outage probability assuming maximum ratio combining (MRC) at the destination. We investigate the effectiveness of the proposed solution by comparing the achieved outage probabilities with those obtained through global numerical search. We find that the simple closed-form solution is almost optimal in many situations of interest and even at low SNR conditions. We observe that power-optimized IR needs significantly less power than power-optimized fixed relaying to achieve the same outage probability at the same effective rate. Finally, we show that our analytical power optimization for IR-MRC systems is also applicable to IR systems with selection diversity and results in almost optimal outage probability.

## 1. INTRODUCTION

Cooperative communication strategies provide distributed spatial diversity in wireless networks and enable energy-efficient transmission of information from remote users (sources) to the desired destination via intermediate relay nodes [1–3]. There exists a variety of ways in which relays can be used in the network to provide distributed diversity. One can broadly categorize cooperative diversity techniques by the type of signal processing performed at the relay for information transfer or by the relay activation method. Two main types of signal processing for information transfer are amplify and forward (AF) and decode and forward (DF) relaying [1]. Examples of relay activation modes include fixed relaying (FR), in which the relay is activated in its designated time regardless of the quality of the source-destination link, and incremental relaying (IR), in which the relay is activated only if the transmission in the source-destination link is in outage [3]. IR can be facilitated by the limited binary feedback from the destination about the success or failure of the transmission in the source-destination or direct link.

It is interesting to compare various performance measures and design aspects of incremental relaying and fixed relaying. For example, one of practically-useful comparative studies would be to an-

alyze power requirements of IR and FR to achieve the same outage probability at the same effective rate. However, for a fair study of power requirements one must first find the optimum power allocation scheme to the source and relay nodes that minimizes the outage probability in each relay activation scheme. Power allocation schemes have been proposed in the literature for the fixed relaying scenario that minimize outage probability or maximize information capacity (see [2, 4–7] and the references therein). Whereas due to the outage-dependant activation of relay nodes in IR, the problem of power allocation in IR is more complicated and has not been investigated so far.

### 1.1. Approach and Contributions

In this paper, we present a novel framework for the optimization of the source/relay power allocation in amplify and forward incremental relaying with the goal to minimize the overall outage probability at the destination. Our framework takes into account the outage-dependent activation of the relay when writing the time and energy constraints. However, solving such power allocation problem is difficult mainly because the outage probability of the direct link is present in the power constraint equation, which itself is non-linearly dependent on the fraction of power allocated to the source. Furthermore and similar to the FR, the overall probability of outage in IR may not have a closed form for signal combining schemes of interest, such as maximum-ratio combining (MRC). In this context, our contributions are as follows:

- We first provide a simple closed-form solution to the power allocation problem in incremental relaying that is based on high signal-to-noise (SNR) approximation of the outage probability in the IR-MRC scheme. In this power allocation scheme, the source power is the solution to a quadratic equation. We investigate the effectiveness of the proposed solution by comparing the achieved outage probabilities with those obtained through global numerical search. We find that the simple closed-form solution is almost optimal in many situations of interest and even at low SNR conditions.
- We then investigate the power savings in using IR compared to FR. We observe that power-optimized IR needs significantly less power than power-optimized FR to achieve the same outage probability at the same effective rate. The power saving can be as high as 8 dB. We note, however, that the optimized power of a relay in IR can be much higher than the average power in the system, which is due to its *sparse* out-

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age dependent activation. We have observed power savings as high as 6 dB in IR even when we limit the relay power to not go above a certain maximum power.

- Finally, we investigate source/relay power optimization in IR when the destination uses selection diversity (SD) instead of MRC. Using exponential approximation of the outage probability in the source-relay-destination link, we propose a closed-form suboptimal solution to power allocation for IR-SD, which turns out to be identical to that in IR-MRC. In our numerical analysis, we then apply the suggested solution to actual outage probability of IR-SD and observe that it is surprisingly very close to the optimal solution found by global search. Moreover, power-optimized IR-SD is still more power efficient than power-optimized FR-MRC.

## 2. SYSTEM MODEL

We consider a wireless amplify and forward relay network with a source, a relay and a destination. We assume that all channels between these nodes are Rayleigh block fading channels and there is perfect CSI available at the relay and destination nodes. We assume block-by-block transmission of data each with duration of  $T$  seconds. During each block transmission, the fading channel stays constant and then changes to an independent value in the next block. The details of transmission are outlined below.

### 2.1. Observation Equations and SNR

In the first time block, the source broadcasts the input signal  $X$  to the destination and relay. The respective received signals at these nodes are given by

$$Y_{s,d} = H_{s,d}X + Z_{d1}, \quad (1)$$

$$Y_{s,r} = H_{s,r}X + Z_r, \quad (2)$$

where the signal power at the source is  $P_S$ .  $Z_{d1}$  and  $Z_r$  are independent zero-mean complex-valued additive white Gaussian noise (AWGN) at the destination and relay, respectively, with variance  $N_0$  each. We assume that the channel fading gain  $H_{s,d}$  follows a complex-valued, zero-mean Gaussian distribution with variance  $\sigma_{H_{s,d}}^2 = 1/(D_{s,d})^a$ , where  $D_{s,d}$  is the distance between the source and the destination and  $a$  is the path-loss exponent.  $H_{s,r}$  is similarly defined. The instantaneous SNR in the source-destination (direct) link is given as

$$\rho_{s,d} = \frac{P_S |H_{s,d}|^2}{N_0}. \quad (3)$$

In FR with variable gain [3], the relay uses the following channel-dependent gain

$$A = \sqrt{\frac{P_R}{P_S |H_{s,r}|^2 + N_0}}, \quad (4)$$

to relay the data to the destination in the second time block, where  $P_R$  is the relay power. Therefore, the received signal at the destination is given by

$$Y_{r,d} = H_{r,d}AY_{s,r} + Z_{d2} \quad (5)$$

$$= H_{r,d}H_{s,r}AX + H_{r,d}AZ_r + Z_{d2}, \quad (6)$$

where  $Z_{d2}$  is AWGN at the destination in the second time block with variance  $N_0$ .  $H_{r,d}$  follows a complex-valued, zero-mean Gaussian

distribution with variance  $\sigma_{H_{r,d}}^2 = 1/(D_{r,d})^a$ , where  $D_{r,d}$  is the distance between the relay and the destination. Using (4), the instantaneous SNR due to source-relay-destination transmission can be written as follows

$$\rho_{s,r,d} = \frac{\rho_{s,r}\rho_{r,d}}{\rho_{s,r} + \rho_{r,d} + 1}, \quad (7)$$

where  $\rho_{s,r}$  and  $\rho_{r,d}$  are individual source-relay and relay-destination SNR's, respectively defined as

$$\rho_{s,r} = \frac{P_S |H_{s,r}|^2}{N_0}, \quad \rho_{r,d} = \frac{P_R |H_{r,d}|^2}{N_0}. \quad (8)$$

Assuming that the destination uses MRC strategy to combine signals from the source and relay, the effective SNR in FR is

$$\rho^{\text{fixed}} = \rho_{s,d} + \rho_{s,r,d}. \quad (9)$$

In IR, the relay transmits the amplified signal to the destination according to (5) *only* when the instantaneous SNR in the direct link  $\rho_{s,d}$  is below a certain threshold  $\rho_{\text{inc}}^{\text{th}}$ , which we will discuss shortly. Such incremental relaying can be facilitated via broadcasting a simple acknowledgement (ACK) or negative ACK (NAK) from the destination back to the source and relay depending on whether  $\rho_{s,d} > \rho_{\text{inc}}^{\text{th}}$  or not. In the case where  $\rho_{s,d} < \rho_{\text{inc}}^{\text{th}}$ , the destination performs MRC to combine the two received signals, which is similar to FR. Therefore, the effective SNR in IR is

$$\rho_{\text{inc}} = \begin{cases} \rho_{s,d} & \text{if } \rho_{s,d} > \rho_{\text{inc}}^{\text{th}}, \\ \rho^{\text{fixed}} = \rho_{s,d} + \rho_{s,r,d} & \text{otherwise.} \end{cases} \quad (10)$$

### 2.2. Information Rates and Outage Probabilities

In FR, the instantaneous information rate is given by

$$I_{\text{fixed}} = \frac{1}{2} \log(1 + \rho^{\text{fixed}}), \quad (11)$$

where the factor of 1/2 accounts for the use of two time blocks for the transmission of the same data. An outage event in FR occurs when  $I_{\text{fixed}}$  falls below a predefined target rate of  $R_{\text{fixed}}$ . We denote the outage probability in FR as  $P_{\text{fixed}}^{\text{out}}$ , which can be written as [3]

$$P_{\text{fixed}}^{\text{out}} = P\left(\frac{1}{2} \log(1 + \rho^{\text{fixed}}) < R_{\text{fixed}}\right) \quad (12)$$

$$= P(\rho^{\text{fixed}} < \underbrace{2^{2R_{\text{fixed}}} - 1}_{\rho_{\text{fixed}}^{\text{th}}}). \quad (13)$$

In IR, depending on whether relay is used or not, the information rate is variable and given as follows

$$I_{\text{inc}} = \begin{cases} \log(1 + \rho_{s,d}) & \text{if } \rho_{s,d} > \rho_{\text{inc}}^{\text{th}}, \\ \frac{1}{2} \log(1 + \rho^{\text{fixed}}) & \text{otherwise.} \end{cases} \quad (14)$$

An initial target rate of  $R_{\text{inc}}$  can be achieved whenever  $\log(1 + \rho_{s,d}) > R_{\text{inc}}$  or  $\rho_{s,d} > 2^{R_{\text{inc}}} - 1$ . However, the target rate reduces to  $R_{\text{inc}}/2$  if the relay is activated. An outage event is declared when neither rates could be sustained. That is [3],

$$P_{\text{inc}}^{\text{out}} = P\left(\log(1 + \rho_{s,d}) < R_{\text{inc}}, \frac{1}{2} \log(1 + \rho^{\text{fixed}}) < \frac{R_{\text{inc}}}{2}\right) \\ = P(\rho^{\text{fixed}} < \underbrace{2^{R_{\text{inc}}} - 1}_{\rho_{\text{inc}}^{\text{th}}}). \quad (15)$$

Based on the above, the average rate in IR is given by

$$R_{\text{inc}}^{\text{eff}} = R_{\text{inc}} P(\rho_{s,d} > 2^{R_{\text{inc}}} - 1) + \frac{R_{\text{inc}}}{2} (P(\rho_{s,d} \leq 2^{R_{\text{inc}}} - 1)).$$

Because the average rate is dependent on the direct link outage probability, a fair comparison of FR and IR is complicated. We note, however, that in high SNR conditions  $P(\rho_{s,d} < 2^{R_{\text{inc}}} - 1)$  may be small and the effective rate can be simply approximated by  $R_{\text{inc}}$ . In the following analysis, we will fix the average rate in FR and IR to the same value  $R_{\text{inc}} = R_{\text{fixed}} = R_{\text{eff}}$ . Referring to (13) and (15), one can deduce that  $\rho_{\text{fixed}}^{\text{th}} > \rho_{\text{inc}}^{\text{th}}$  as a result of fixing  $R_{\text{inc}} = R_{\text{fixed}} = R_{\text{eff}}$ . That is, FR needs to maintain a higher SNR than IR to achieve the same effective rate. Our aim is to quantify FR and IR power requirements.

Furthermore, for a fair comparison of the two relaying schemes, we will fix the average power of each block to  $P$ . In the next section, we will first review the traditional FR power allocation scheme and then present the power allocation constraint for IR and discuss its solution for minimizing outage probability.

### 3. POWER ALLOCATION SCHEMES

#### 3.1. Fixed Relaying

The FR power allocation constraint is relatively simple due to the fact that the relay is used regardless of the SNR condition in the direct link. The conservation of time and energy dictates that

$$TP_S + TP_R = 2TP \Rightarrow P_S + P_R = 2P \quad (16)$$

Some authors have considered optimal power allocation in FR based on various system performance criteria. For example, [4] proposed a closed-form suboptimal power allocation solution based on high-SNR approximation of the outage probability.

#### 3.2. Incremental Relaying

Due to the probabilistic involvement of the relay in IR, special attention is needed in deriving the power constraint. As we will see below, the power constraint depends on the outage probability in the direct link. Let us denote this probability as  $P_{\text{dir}}^{\text{out}} = P(\rho_{s,d} < 2^{R_{\text{inc}}} - 1) = P(\rho_{s,d} < \rho_{\text{inc}}^{\text{th}})$ . Therefore, we can see that, on average, the relay is used  $P_{\text{dir}}^{\text{out}}$  percent of the time and is silent in other times. So we can write the following statistical power constraint based on the relay activity and conservation of time and energy:

$$TP_S + TP_{\text{dir}}^{\text{out}} P_R = (1 + P_{\text{dir}}^{\text{out}})TP \Rightarrow \quad (17)$$

$$P_S + P_{\text{dir}}^{\text{out}} P_R = (1 + P_{\text{dir}}^{\text{out}})P. \quad (18)$$

From (18), we note the difficulty in trying to derive the optimal power allocation for the source and relay. This is because the power constraint depends on the direct link outage probability which, in turn, depends on the power allocated to the source in a non-linear manner. This becomes even more complicated if the aim of power allocation is to minimize the overall outage probability. This is due to the fact that the distribution of overall SNR  $\rho_{\text{fixed}}$  in (15) is complicated in general. To the best of our knowledge, IR power allocation under power constraint (18) has not been considered before in the literature nor compared with that in FR.

In what follows, we will make use of upper bounds on  $\rho_{s,r,d}$  and high-SNR approximation of the outage probability (15) to propose a suboptimal solution for power allocation in IR under power constraint (18). Later in Section 4 we will show that the proposed power allocation scheme works very well and the outage probabilities are very close to those found using global search.

#### 3.3. High-SNR Closed-Form Solution

First, using the results in [4], we can upper bound the average SNR in the source-relay-destination link given in (7) as follows

$$E\{\rho_{s,r,d}\} = E\left\{\frac{\rho_{s,r}\rho_{r,d}}{\rho_{s,r} + \rho_{r,d} + 1}\right\} \leq \frac{\bar{\rho}_{s,r}\bar{\rho}_{r,d}}{\bar{\rho}_{s,r} + \bar{\rho}_{r,d}} = \bar{\rho}_{s,r,d}^{\text{UB}}, \quad (19)$$

where  $\bar{\rho}_{s,r} = E\{\rho_{s,r}\}$  and  $\bar{\rho}_{r,d} = E\{\rho_{r,d}\}$  refer to the average SNR in the source-relay and relay-destination links, respectively. Shortly, we will use the upper bound  $\bar{\rho}_{s,r,d}^{\text{UB}}$  in the optimization of the outage probability.

Comparing the outage probability of IR (15) with that of FR (13), we find that apart from possibly a different SNR threshold, they essentially have similar characteristics because of their dependence on  $\rho_{\text{fixed}}$ . Hence, the high-SNR approximation proposed in [3] for FR also applies to IR. More specifically, at high SNR we have

$$P_{\text{inc}}^{\text{out}} \sim \frac{(\rho_{\text{inc}}^{\text{th}})^2}{2\bar{\rho}_{s,d}\bar{\rho}_{s,r,d}^{\text{UB}}}, \quad (20)$$

where  $\bar{\rho}_{s,d} = E\{\rho_{s,d}\}$  refers to the average SNR in the direct or source-destination link. Therefore, for a fixed  $\rho_{\text{inc}}^{\text{th}}$ , minimization of outage probability is equivalent to maximization of the product  $\bar{\rho}_{s,d}\bar{\rho}_{s,r,d}^{\text{UB}}$ . Referring to (3) and (8), we can write this product as

$$\bar{\rho}_{s,d}\bar{\rho}_{s,r,d}^{\text{UB}} = b_0 P_S \frac{b_1 P_S b_2 P_R}{b_1 P_S + b_2 P_R}, \quad (21)$$

where for brevity of derivations, we have defined  $b_0 = \sigma_{H_{s,d}}^2/N_0$ ,  $b_1 = \sigma_{H_{s,r}}^2/N_0$ , and  $b_2 = \sigma_{H_{r,d}}^2/N_0$ . Now using the power constraint (18), we can write the relay power in terms of the source power and direct link outage probability as follows

$$P_R = (1 + \frac{1}{P_{\text{dir}}^{\text{out}}})P - \frac{P_S}{P_{\text{dir}}^{\text{out}}}. \quad (22)$$

Since the instantaneous SNR in the direct link follows an exponential distribution, it can be easily shown that  $P_{\text{dir}}^{\text{out}} \sim \rho_{\text{inc}}^{\text{th}}/b_0 P_S$  at high SNR. Upon substituting this approximation in (22) we obtain

$$P_R \sim (1 + \frac{b_0 P_S}{\rho_{\text{inc}}^{\text{th}}})P - \frac{b_0 P_S^2}{\rho_{\text{inc}}^{\text{th}}} \sim \frac{b_0 P_S}{\rho_{\text{inc}}^{\text{th}}}(P - P_S). \quad (23)$$

Now if we use (23) in (21) and take the derivative with respect to  $P_S$ , we can find the optimal source power  $P_S$ , which is the solution to the following quadratic equation

$$\frac{2b_0 b_2}{\rho_{\text{inc}}^{\text{th}}} P_S^2 - \left(\frac{4b_0 b_2}{\rho_{\text{inc}}^{\text{th}}} P + 3b_1\right) P_S + 2b_1 P + \frac{2b_0 b_2}{\rho_{\text{inc}}^{\text{th}}} P^2 = 0, \quad (24)$$

which can be easily found in closed-form.

#### 3.4. Discussion on Selection Diversity

It is now worth discussing IR power allocation when the destination employs selection diversity (SD) instead of MRC. SD is clearly a simple option as it removes the need for co-phasing of the signals transmitted by the source and relay at the destination, which can require complicated hardware for channel estimation and phase-tracking. We note that SD can be a particularly reasonable choice in IR because the complicated co-phasing hardware would be used only  $P_{\text{inc}}^{\text{out}}$  percent of time when the relay is active. In IR-SD, the

destination simply discards the signal received directly from source whenever its SNR falls below the given threshold and only works with the signal from relay in the next time block.

The question that we would like to answer is what is the IR-SD power allocation scheme that minimizes outage probability. First, we note that in IR-SD the outage probability is as follows

$$\begin{aligned} P_{\text{inc,sel}}^{\text{out}} &= P\left(\log(1 + \rho_{s,d}) < R_{\text{inc}}, \frac{1}{2} \log(1 + \rho_{s,r,d}) < \frac{R_{\text{inc}}}{2}\right) \\ &= P\left(\rho_{s,d} < \rho_{\text{inc}}^{\text{th}}\right) P\left(\rho_{s,r,d} < \rho_{\text{inc}}^{\text{th}}\right), \end{aligned} \quad (25)$$

where the second equality follows from independence of direct and source-relay-destination links. Moreover, using the results in [8] for selection diversity we can write the second term in (25) as

$$\begin{aligned} P\left(\rho_{s,r,d} < \rho_{\text{inc}}^{\text{th}}\right) &= 1 - 2\sqrt{\frac{(\rho_{\text{inc}}^{\text{th}})^2 + \rho_{\text{inc}}^{\text{th}}}{\bar{\rho}_{s,r} + \bar{\rho}_{r,d}}} \\ &\cdot \exp\left(-\rho_{\text{inc}}^{\text{th}} \frac{\bar{\rho}_{s,r} + \bar{\rho}_{r,d}}{\bar{\rho}_{s,r} \bar{\rho}_{r,d}}\right) K_1\left(2\sqrt{\frac{(\rho_{\text{inc}}^{\text{th}})^2 + \rho_{\text{inc}}^{\text{th}}}{\bar{\rho}_{s,r} + \bar{\rho}_{r,d}}}\right) \end{aligned} \quad (26)$$

Due to the complicated nature of the above outage probability and power constraint (18), we have been unable to derive any closed-form optimal or suboptimal solution that minimizes (25) directly. However, similar to [9], we use the following argument to approximate  $\rho_{s,r,d}$  with an exponential distribution. First we note that at high SNR, one can write  $\rho_{s,r,d}$  as

$$\rho_{s,r,d} \sim \frac{1}{\frac{1}{\rho_{s,r}} + \frac{1}{\rho_{r,d}}}. \quad (27)$$

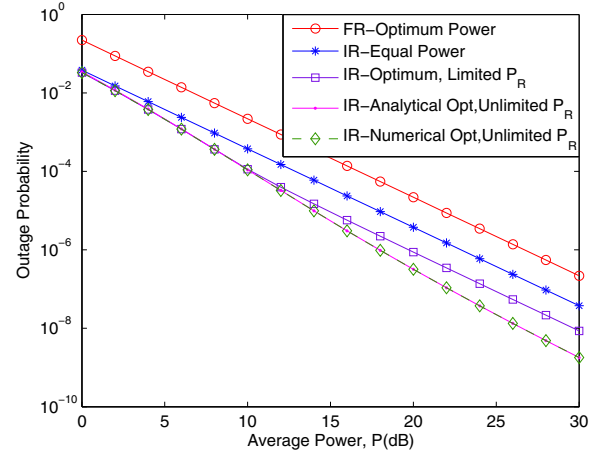
Then for the case where the SNR in one of the hops is much stronger than the other ( $\rho_{s,r} \gg \rho_{r,d}$  or *vice versa*), one can argue that the distribution of  $\rho_{s,r,d}$  is very close to exponential. Therefore, we simply approximate  $\rho_{s,r,d}$  with an exponential distribution with an upper bound on its mean given by (19). Using the above argument, we simply write the overall IR-SD outage probability as

$$\begin{aligned} P_{\text{inc,sel}}^{\text{out}} &\sim \left(1 - \exp\left(-\frac{\rho_{\text{inc}}^{\text{th}}}{\bar{\rho}_{s,d}}\right)\right) \left(1 - \exp\left(-\frac{\rho_{\text{inc}}^{\text{th}}}{\bar{\rho}_{s,r,d}}\right)\right) \\ &\sim \frac{(\rho_{\text{inc}}^{\text{th}})^2}{\bar{\rho}_{s,d} \bar{\rho}_{s,r,d}^{\text{UB}}}. \end{aligned} \quad (28)$$

Comparing (28) for IR-SD with (20) for IR-MRC, we find that these outage probabilities only differ in a factor of 2 and hence, minimization of (28) is equivalent to the minimization of (20) for which we have already provided a power allocation solution in (24). In Section 4 we will show that using an approximate formula for IR-SD power allocation will result in outage probabilities that are very close to the optimal solution found using global search.

#### 4. NUMERICAL RESULTS

In this section, we study the efficiency of our novel power allocation scheme in IR in achieving the same outage probability and effective rate as power-optimized FR systems. We assume that the source, relay and destination are placed on a straight line and affected by the same shadowing environment [5]. This assumption is only needed for shadowing effect that has not been considered in this model. In other words, it does not affect the accuracy of our analysis or performance of the power allocation. The normalized distance between



**Fig. 1.** The outage probability of IR with different power allocation schemes and FR with optimum power vs. average power per block,  $D_{s,r} = 0.25$ ,  $D_{r,d} = 0.75$ .

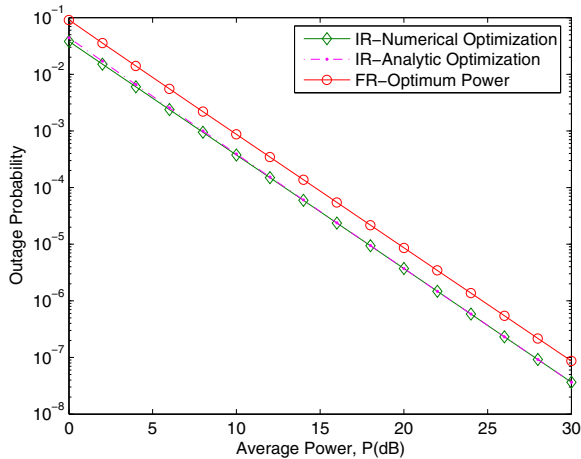
the source and the destination is  $D_{s,d} = 1$ . Fig. 1 shows the outage probability for the case that the relay is closer to the source, *i.e.*, the distance between the source and the relay is  $D_{s,r} = 0.25$  and the relay-destination distance is  $D_{r,d} = 0.75$ . The path-loss exponent is  $\alpha = 3$ . Average power per block is  $P$  and varies from 0 to 30 dB. Noise variance  $N_0$  is normalized to unity and effective rate is  $R_{\text{eff}} = 0.5$  bits/sec/Hz. Similar system parameters have been used in [4, 5, 10]. In this figure (Fig. 1), IR with our power allocation introduced in Section 3.3 has been compared to FR with optimum power allocation (numerical optimization) to minimize outage probability under power constraint in Section 3.1. Optimized power IR substantially outperforms FR with the same effective rate, *i.e.*,  $R_{\text{inc}} = R_{\text{fixed}} = R_{\text{eff}}$ <sup>1</sup>. For example, for outage probability of  $10^{-6}$ , there is 8 dB difference between FR and IR systems (the first and the last curves). In addition, we observe that although some simplifying high-SNR approximations have been applied to find a closed-form expression for power allocation, this analytic approach is successfully matched with numerical optimization.

The power efficiency of optimized IR is remarkable. However, we notice that at high SNR because of low value of the outage probability of the direct link that appears at the denominator of (22), the power of the relay can be much greater than average power ( $P$ ). Our numerical results show that for the case that the relay is close to the source,  $P_R$  can be more than 100 times greater than  $P$ . Therefore, we consider the maximum power of  $P_{\text{max}}$ , *e.g.*,  $P_{\text{max}} = 5P$  for the relay. Note that the power of the source can not be greater than  $P$  due to our power constraint solution. If the power of the relay is greater than  $P_{\text{max}}$ , then we clip the power to  $P_{\text{max}}$  and compute the power of the source as follows

$$\begin{aligned} P_S + P_{\text{dir}}^{\text{out}} P_R &= (1 + P_{\text{dir}}^{\text{out}})P, \quad P_{\text{max}} \triangleq L_{\text{max}}P \quad (29) \\ \Rightarrow P_S &= \frac{1}{2} \left[ P + \sqrt{P^2 - 4(L_{\text{max}} - 1) \frac{\rho_{\text{inc}}^{\text{th}} P}{b_0}} \right] \end{aligned} \quad (30)$$

As it can be observed in Fig. 1, the performance of the system in limited power case ( $L_{\text{max}} = 5$ ) is worse than unlimited one, however it still has a very good performance in comparison with equal

<sup>1</sup>For incremental relaying system, the effective rate is a little bit less than  $R_{\text{eff}}$ , however it is negligible especially at high SNR [3].



**Fig. 2.** The outage probability of IR with power allocation scheme given in (24) and FR with optimum power vs. average power per block,  $D_{s,r} = 0.75$ ,  $D_{r,d} = 0.25$ .

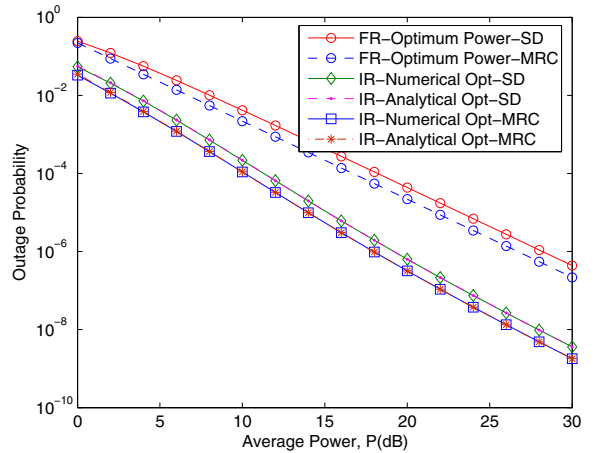
power case and FR system. For example, for the outage probability of  $10^{-6}$ , there is still 6 dB difference between FR and IR systems with limited power. Also, there is about 3 dB difference between the outage performance of equal power and optimum power schemes. Note that for reverse condition that the relay is closer to the destination, the numerical result shows that limiting the power of the relay does not affect the performance. The reason is that for this case, the optimum value of  $P_R$  derived from (22) is less than  $P_{\max}$ .

Fig. 2 shows the outage probabilities for the opposite case, *i.e.*, the source-relay distance is  $D_{s,r} = 0.75$  and the relay-destination distance is  $D_{r,d} = 0.25$ . In this case, the difference between fixed relaying and incremental relaying for outage probability of  $10^{-6}$  is about 2 dB. In other words, the improvement of our power allocation is less than before. However, it still needs so much less power (63%) in comparison with FR scheme.

In Fig. 3, we have applied our power allocation solution on the selection diversity receiver and compared it with previously-considered MRC system. As we have shown in Section 3.4, the outage probability in SD is twice that in MRC at high SNR. However, this figure shows that even at low SNR the ratio more or less holds. Note that the final results of power allocation is applied on the exact expression of outage probability of the system given in (25). Therefore, our analytical approach to find optimum power solution for (28) is successfully matched with optimum power from numerical optimization of (25) using exact exponential distribution for outage probability of the direct link and (26) for the outage of the relay link. As a result, our high-SNR solution can be used for both MRC and SD systems.

## 5. CONCLUSIONS

We have compared the performance of IR and FR systems using new power allocation scheme. We have considered the outage-dependent activation of the relay for our power constraint. It was shown that power-optimized IR substantially outperforms power-optimized FR at the same effective rate. In addition, our analytical power optimization for IR-MRC systems is also applicable to IR systems with selection diversity and results in almost optimal outage probability.



**Fig. 3.** The outage probability of IR and FR for MRC and SD schemes vs. average power per block,  $D_{s,r} = 0.25$ ,  $D_{r,d} = 0.75$ .

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