

# NON SPHERICAL MICROPHONE ARRAY STRUCTURES FOR 3D BEAMFORMING AND SPHERICAL HARMONIC ANALYSIS

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## ABSTRACT

Decomposition of a soundfield into spherical harmonics is a fundamental problem in acoustic signal processing. This paper shows how to design non-spherical microphone array structures in 3D space to decompose a soundfield into spherical harmonic components. We use the mode limiting property of the Bessel functions and zeros of the associated Legendre functions together with orthogonality of the exponential functions over circles to construct an array of parallel circles of microphones. The result provides flexible design guidelines to construct 3D arrays than traditional spherical arrays. A simple beamforming example is given to verify the result.

**Index Terms**— Spherical array of microphones, spherical harmonics, circular array, beamforming, soundfield

## 1. INTRODUCTION

Array of microphone structures, which are capable of processing acoustic signals to extract useful spatial information of the three dimensional surroundings, are important in plethora of applications such as beamforming, direction of arrival estimation, spatial soundfield recording etc. The spherical harmonic analysis of the 3D acoustic field, which is commonly known as modal analysis [1] technique, has been shown to be a useful tool to design signal processing algorithms for these applications [2–5]. The spherical array of microphones are suited in decomposing 3D acoustic field into spherical harmonic components [2, 6–8], which could then be processed as necessary to form beams in desired directions, to estimate source locations, or to record spatial sounds. In this paper, we show an alternative 3D structure consists of circular arrays that is different to a traditional spherical array structure, to decompose acoustic field into spherical harmonic coefficients.

Meyer and Elko [9] proposed a method to use circular arrays of microphones on the x-y plane together with a centre microphone at the origin to extract spherical harmonic coefficients. This is a novel use of circular arrays as typical use of circular arrays are to decompose a sound field in to cylindrical harmonics, which is more suited for height invariant 2D soundfields. Although, Meyer’s work give some flexibility in controlling vertical spatial response, fundamentally a 2D array on a x-y plane is not able to determine half of the spherical harmonic coefficients. We show this fact in Section 3. In this paper, we investigate the spherical harmonic decomposition and propose a systematic way to build a 3D flexible array structure consists of circular arrays in a set of parallel planes to x-y plane to decompose a given 3D acoustic field into spherical harmonic components.

## 2. SOUNDFIELD ANALYSIS

### 2.1. Spherical harmonic expansion

A soundfield at a point  $(r, \theta, \phi)$  in a source free region can be expressed in terms of spherical harmonic expansion as

$$S(r, \theta, \phi; k) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \alpha_{nm}(k) j_n(kr) Y_{nm}(\theta, \phi) \quad (1)$$

where  $m$  and  $n$  ( $\geq 0$ ) are integers,  $\alpha_{nm}(k)$  are the spherical harmonic coefficients of the soundfield,  $k = 2\pi f/c$  is the wavenumber,  $f$  is the frequency,  $c$  is the speed of sound,  $j_n(\cdot)$  are the spherical Bessel functions of order  $n$ , and spherical harmonics

$$Y_{nm}(\theta, \phi) = \sqrt{\frac{2n+1}{4\pi} \frac{(n-|m|)!}{(n+|m|)!}} P_{n|m|}(\cos \theta) e^{jm\phi} \quad (2)$$

which is defined in terms of Associated Legendre functions  $P_{n|m|}(\cdot)$  and exponential functions. Knowing the soundfield over angles, harmonic coefficients can be calculated using

$$\alpha_{nm}(k) = \frac{1}{j_n(kr)} \int_0^{2\pi} \int_0^{\pi} S(r, \theta, \phi; k) Y_{nm}^*(\theta, \phi) \quad (3)$$

provided  $j_n(kr) \neq 0$ . For convenience, we express (1) in terms of normalized associated Legendre and exponential functions as

$$S(r, \theta, \phi; k) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \alpha_{nm}(k) j_n(kr) \mathcal{P}_{n|m|}(\cos \theta) E_m(\phi) \quad (4)$$

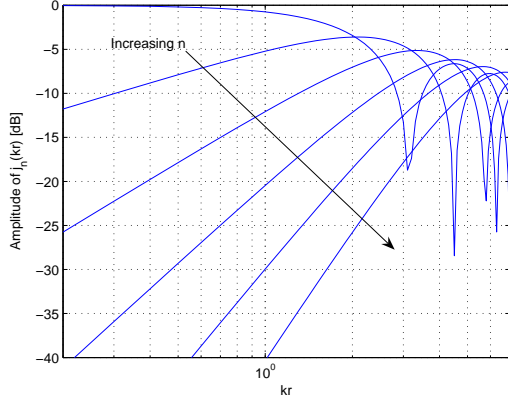
where  $E_m(\phi) \triangleq (1/\sqrt{2\pi}) e^{jm\phi}$  and

$$\mathcal{P}_{n|m|}(\cos \theta) \triangleq \sqrt{\frac{2n+1}{2} \frac{(n-|m|)!}{(n+|m|)!}} P_{n|m|}(\cos \theta), \quad (5)$$

which form orthonormal basis sets in azimuth  $\phi \in [0, 2\pi)$  and elevation  $\theta \in [0, \pi]$ .

### 2.2. Truncation

The representation series (4) can be safely truncated [10] to a finite number using the properties of the Bessel function (see Fig. 1) provided that all sources are located outside the region of interest (e.g., aperture of the array structure). Let  $R$  is the radius of a spherical region of interest, then the soundfield inside this sphere can be represented by (4) with summation over  $n$  truncated to  $N = \lceil ekR/2 \rceil$  terms [10]. We approximate this bound to  $N = \lceil kR \rceil$  to use in this paper.



**Fig. 1:** Magnitude of the spherical Bessel functions of order  $n = 0, 1, 2, 3, 4, 5$  in dB showing the characteristics as a function of the argument.

### 2.3. Soundfield Coefficients

Suppose the soundfield is restricted to have only first  $N + 1$  coefficients due to finite dimension of the region of interest. Then, we can see from (4) that there are total of  $(N + 1)^2$  coefficients to be determined. Table 1 depicts the growth of number of coefficients to be determined as order  $n$  grows with modes ranging from  $-n$  to  $n$ . The soundfield coefficients can be estimated by sampling the space

$m \setminus n$	0	1	2	...	$N$
$N$					$\alpha_{NN}$
2			$\alpha_{22}$		$\vdots$
1		$\alpha_{11}$	$\alpha_{21}$		$\vdots$
$m = 0$	$\alpha_{00}$	$\alpha_{10}$	$\alpha_{20}$	...	$\alpha_{N0}$
-1		$\alpha_{1(-1)}$	$\alpha_{2(-1)}$		
-2			$\alpha_{2(-2)}$		$\vdots$
$-N$					$\alpha_{N(-N)}$

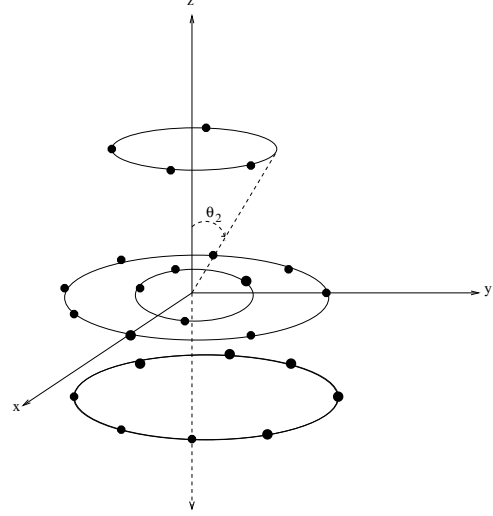
**Table 1:** Soundfield coefficients arranged with order  $n$  and mode  $m$ .

using an array of sensors. Spherical microphone arrays approximate the analysis equation (3) by a sum of samples of signal taken over the spherical surface to perform this tasks. However, there are known limitation of the spherical arrays [7] such as strict orthogonality condition and inflexibility with the sensor geometry. In this paper we develop an alternative array structure to estimate soundfield coefficients.

### 3. CIRCULAR APERTURE

In this section, we investigate the soundfield on a circle on a plane parallel to the x-y plane. Let  $S(r_q, \theta_q, \phi; k)$  be the the soundfield on a circle given by  $\theta = \theta_q$  and  $r = r_q$ , where  $\theta_q$  and  $r_q$  are suitably chosen constants. We use (4) to write

$$S(r_0, \theta_0, \phi; k) = \sum_{n=0}^N \sum_{m=-n}^n \alpha_{nm}(k) j_n(kr_q) \mathcal{P}_{n|m|}(\cos \theta_q) E_m(\phi) \quad (6)$$



**Fig. 2:** Circular array structure for 3rd order spherical harmonic decomposition

where  $N = \lceil kr_q \rceil$  is due to the natural truncation property (see Section 2.2). We multiply (6) by  $E_{-m}(\phi)$  and integrate with respect to  $\phi$  over  $[0, 2\pi)$  to get

$$\sum_{n=|m|}^N \alpha_{nm}(k) j_n(kr_q) \mathcal{P}_{n|m|}(\cos \theta_q) = \int_0^{2\pi} S(r_q, \theta_q, \phi; k) E_{-m}(\phi) d\phi. \quad (7)$$

We have following comments regarding (7):

- Left hand side (LHS) of (7) is a weighted sum of soundfield coefficients along a row for a given  $m$  in Table 1.
- Equation (7) can be evaluated for  $m = -N, \dots, N$ , where the truncation number  $N$  dependent on the radius  $r_q$  of the circle.
- For a given circle  $(r_q, \theta_q)$  some of the spherical harmonics could be zero if either  $j_n(kr_q) = 0$  or  $\mathcal{P}_{n|m|}(\cos \theta_q) = 0$ . Thus, care need to be taken while using (7) for coefficient calculations. Figures 3 and 4 depict the magnitude of normalized associated Legendre functions  $\mathcal{P}_{n|m|}(\cos \theta)$  in dBs.
- The main contribution of this paper is to extract spherical harmonic coefficients  $\alpha_{nm}(k)$  by exploiting (7) from a number of carefully placed circles on different  $(r_q, \theta_q)$  together with properties of Spherical Bessel functions and associated Legendre functions.

Suppose we are interested in designing a  $N$ th order microphone array to estimate  $(N + 1)^2$  spherical harmonic coefficients. Let there are  $Q \geq (N + 1)$  circles of microphones located on planes given by  $(r_q, \theta_q)$ ,  $q = 1, \dots, Q$ . Also choose  $r_q$  such that  $n = \lceil kr_q \rceil$ , where  $n = 0, \dots, N$ , i.e., each  $r_q$  corresponds to a specific  $n$ . This condition is strictly not necessary but significantly simplifies the estimation of  $\alpha_{nm}$  as smaller circles do not contain higher order harmonic coefficients; e.g., a single sensor at the origin only contains  $\alpha_{00}$ .

By writing (7) for a specific  $m$  for all applicable circles, we have

$$\mathbf{J}_m \boldsymbol{\alpha}_m = \mathbf{a}_m, \text{ for } m = -N, \dots, N \quad (8)$$

where  $\mathbf{J}_m =$

$$\begin{bmatrix} j_{|m|}(kr_1)\mathcal{P}_{|m||m|}(\cos\theta_1) & \cdots & j_N(kr_1)\mathcal{P}_{N|m|}(\cos\theta_1) \\ \vdots & \ddots & \vdots \\ j_{|m|}(kr_Q)\mathcal{P}_{|m||m|}(\cos\theta_Q) & \cdots & j_N(kr_Q)\mathcal{P}_{N|m|}(\cos\theta_Q) \end{bmatrix}, \quad (9)$$

$\boldsymbol{\alpha}_m = [\alpha_{|m|m}, \alpha_{|m+1|m}, \dots, \alpha_{Nm}]^T$ , and  $\mathbf{a}_m = [a_{1m}, \dots, a_{Qm}]^T$ , with  $a_{qm} = \int_0^{2\pi} S(r_q, \theta_q; k) E_{-m}(\phi) d\phi$ .

If  $(r_q, \theta_q)$ ,  $q = 1, \dots, Q$  are chosen such that  $\mathbf{J}_m$  has a valid Moore-Penrose inverse  $\mathbf{J}_m^+$ , then  $\boldsymbol{\alpha}_m$  can be calculated for each  $m$  by solving (8) in the least squares sense as

$$\boldsymbol{\alpha}_m = \mathbf{J}_m^+ \mathbf{a}_m. \quad (10)$$

However, we can be bit more creative in placing circles to exploit the underline structure of the wave propagation rather than relying on the ability of the least squares to do the job. To do this, we inspect (8) for specific values of  $(r_q, \phi_q)$  and  $m$ :

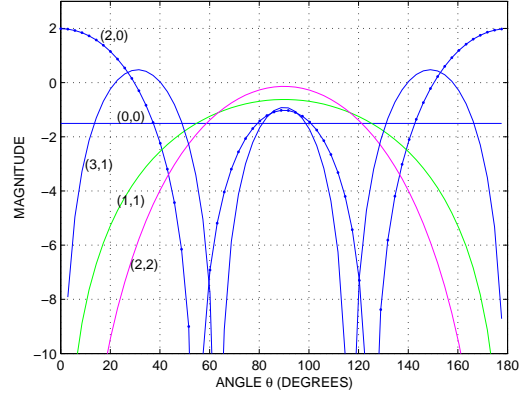
- For a single sensor at the origin,  $(r_q, \phi_q) = (0, 0)$ , only available mode is  $m = 0$ , thus  $\alpha_{00} = a_{00}(0, 0)/\mathcal{P}_{0|0|}(1)$ .
- For  $\theta = \pi/2$ ,  $\mathcal{P}_{n|m|}(\cos(\pi/2)) = 0$  if  $n + |m|$  is odd. This can be seen in Fig. 4 which depicts the normalized associated Legendre functions up to order 3 for odd  $n + |m|$  values. Also note that from Fig. 3, for  $n + |m|$  even values, normalized associated Legendre functions do not greatly attenuate the corresponding harmonic coefficients at  $\theta = \pi/2$ . In this paper, we use this property in our design example to illustrate the power of this technique.
- When  $m = N$ , (8) reduces to a single equation with one unknown  $\alpha_{NN}$ . However, if we use this equation alone to calculate  $\alpha_{NN}$  there may be significant errors involved as there could be contribution from  $\alpha_{(N+1)N}$  in this equation. Note from Fig. 1, dB reduction from first mode to second mode at  $r$  (with  $1 = \lceil kr \rceil$ ) is 7dB, from 2 to 3 at the circle with  $2 = \lceil kr \rceil$  is about 5dB, and so on. Thus, to improve the accuracy of this calculation, we may need measurements from an additional circle. However, for a set of circles on the  $\theta = \pi/2$  plane, this is not a problem since there is no contribution from the next mode because its sum of order and mode is odd.
- Also note that any sensor on the z-axis only contains  $m = 0$  coefficients.

Above are some of the properties we can observe from (7) together with the characteristics of spherical Bessel functions (Figure 1 and normalized associated Legendre functions (Figures 3 and 4). We assert that a design engineer can use these properties to construct a 3D array structure to estimate spherical harmonic coefficients of spatial soundfields, without relying on spherical array structure as in the literature [2, 6–8].

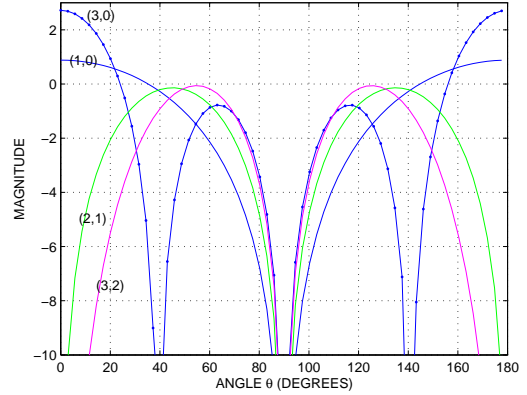
Until now we showed how to calculate sound field coefficients given the soundfield on number of circular apertures. In practice, we can not obtain soundfield at every point on these circles. In the following section, we show how to use only samples of soundfield on these circles. This will enable us to give design guidelines to build practical microphone arrays.

#### 4. IMPLEMENTATION

Let the design objective is to construct an array capable of decomposing soundfield coefficients up to  $N$  modes. We use a set of circular arrays to calculate the spherical harmonic coefficients.



**Fig. 3:** Magnitude of the normalized associate Legendre functions  $\mathcal{P}_{n|m|}(\cos\theta)$  in dB, where the addition of order  $n$  and mode  $m$  are even:  $(n, |m|) = (0, 0); (2, 0); (1, 1); (2, 2); (3, 1); (3, 3)$



**Fig. 4:** Magnitude of the normalized associate Legendre functions  $\mathcal{P}_{n|m|}(\cos\theta)$  in dB, where the addition of order  $n$  and mode  $m$  are odd:  $(n, |m|) = (1, 0); (2, 1); (3, 0); (3, 2)$

##### 4.1. Sampling of circles

For the circular aperture at  $r_q$ , we need to evaluate the integral in (7) with a summation since in practice we can only have a finite number of samples of  $S(r_q, \theta_q, \phi; k)$ . For this radius, if the field is limited to  $n$  orders (i.e.  $n = \lceil kr_q \rceil$ ), the maximum mode  $m$  involved is  $n$ . Thus,  $S(r_q, \theta_q, \phi; k)$  is mode limited to  $n$ , i.e., it contains terms with  $e^{jm\phi}$  with  $m = 0, \dots, n$ . According to Shannon's sampling theorem for periodic functions,  $S(r_q, \theta_q, \phi; k)$  can be reconstructed by its samples over  $[0, 2\pi]$  with at least  $2n + 1$  samples. Hence we can approximately replace the right hand side (RHS) of (7) by

$$\text{RHS of (7)} \approx \frac{2\pi}{V_n} \sum_{v=1}^{V_n} S(r_q, \theta_q, \phi_v; k) E_m(\phi_v), \quad (11)$$

where  $V_n \geq 2n + 1$ . Thus, to calculate coefficients of all orders and modes up to  $N$ th order, we need at least  $(N + 1)^2$  microphones over  $N + 1$  circles. That is, a single sensor at the origin, 3 sensors on a the next circle, followed by 5 sensors, and ends with  $2N + 1$  sensors on the  $N + 1$ th circle.

## 4.2. Placing circles

By following the description given in Section 2.3, we place alternative circles on the x-y plane (i.e.  $\theta_q = 0$ ) corresponding to  $n = \lceil kr_q \rceil$  with  $n = 1, 3, 5, \dots$ . Then, we can write (8) as  $\mathbf{J}_m^e \boldsymbol{\alpha}_m^e = \mathbf{a}_m$ , where  $\mathbf{J}_m^e$  and  $\boldsymbol{\alpha}_m^e$  contain elements corresponding to even  $n + |m|$ . This equation could be solved to find coefficients  $\alpha_{nm}$  with  $n + |m|$  even. Observe from Fig. 4 that we can place the second set of circles corresponding to  $n = 2, 4, \dots$  at  $\theta_q \in \{[50^\circ : 70^\circ], [110^\circ, 130^\circ]\}$  to calculate the rest of the coefficients. In this case, we can rewrite (8) as  $\mathbf{J}_m^o \boldsymbol{\alpha}_m^o = \mathbf{a}_m - \mathbf{J}_m^e \boldsymbol{\alpha}_m^e$  where  $\mathbf{J}_m^o, \boldsymbol{\alpha}_m^o$  and  $\mathbf{J}_m^e, \boldsymbol{\alpha}_m^e$  can be defined accordingly to represent odd and even  $n + |m|$  respectively.

## 4.3. Broadband

To make the array work for broadband without adding infinitely many circles for each frequency, we need to reuse existing set of circles for higher frequencies. Suppose the desired frequency band is  $[k_l, k_u]$ . Let  $r_n(k_l)$  be the radial distance to the  $n$ th circle from the origin corresponding to frequency  $k_l$ , which we use to calculate coefficients of order  $n$ . Since we use the constraint  $n = \lceil kr_n \rceil$  to choose  $r_n(k_l)$ , it is easy to see that  $r_n(k_l) = r_{sn}(s \times k)$ , where  $s$  is a positive integer. That is, we can use the same circle for realizing higher order coefficients of the integer multiplier of the initial frequency. However, we need more sensors on the circle to calculate higher orders. This concept could be considered as similar to nested arrays in the line array literature for broadband. We stop short of giving detail broadband design in this paper due to space constraints.

## 5. SIMULATIONS

To illustrate our theoretical array design, we show an example of 3rd order ( $N = 3$ ) array of circular arrays of microphones in 3D beamforming. Let the design frequency be 1.5KHz and the speed of sound propagation  $c = 340\text{ms}^{-1}$ . First, we place a microphone at the origin to estimate  $\alpha_{00}$ . Next we have two circular arrays on the x-y plane ( $\theta_q = 0$ ) with radii  $1/k = 3.6\text{cm}$  and  $3/k = 10.8\text{cm}$  respectively. Two circles have 4 and 8 uniformly separated microphones, which are one more than the minimum required. The output from these microphones on the x-y plane can be used to calculate, all  $\alpha_{nm}$  coefficients where  $n + |m|$  is an even number (up to  $n = 3$ ). Finally, we place two more circles with  $(r_3, \theta_3) = (1/k, 0.9)$  and  $(r_4, \theta_4) = (3/k, \pi - 0.9)$  with 4 and 8 microphones respectively. We use the output of the second set of microphones together with already estimated coefficients to calculate the rest of the coefficient set as outlined in Section 4.2. Figure 2 shows an example array geometry.

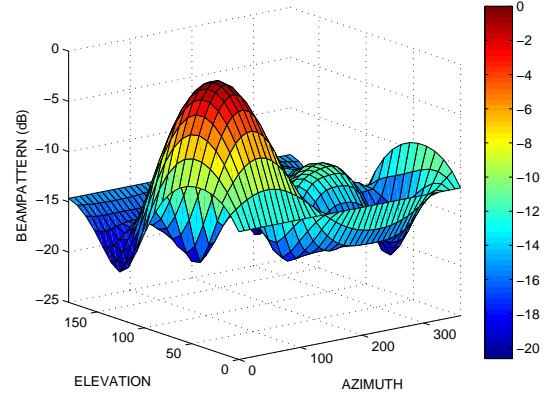
By weighting each estimated coefficient by

$$(i^{-n}/(2n+1))\mathcal{P}_{n|m}|(\cos \theta_\ell)E_m(\phi_\ell)$$

and summing together will give a beamformer in the look direction  $(\theta_\ell, \phi_\ell)$ . Figure 5 depicts the response of the above array structure steered to the look direction of  $(\pi/3, \pi/3)$ .

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**Fig. 5:** Response of the Beamformer example in Section 5 where the look direction  $(\pi/3, \pi/3) = (60^\circ, 60^\circ)$ .

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