Spatial Decorrelation Receiving technique for Frequency Selective SIMO Channels

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***Abstract*—In multi-antenna system, the correlations between the receiver antennas degrades the systems performance. In this paper, we propose a spatial decorrelation receiving technique in frequency-selective SIMO channels. A spatial-to-modal matrix is placed after the SIMO channel output to remove the correlations between the receiver antennas caused by the limited antenna separations. Simulation results of this newly proposed spatial decorrelation design demonstrate favourable BER improvement and better system performance.**

I. Introduction

Multi-antenna communication systems have drawn increase attention due to the higher capacity and promising improved performance. For frequency-selective fading channels, which generally arise in broadband wireless systems, most of the existing work [1], [2], [3] on channel equalization/ estimation are based on the assumption of independent fading between the transmitter-receiver antenna pairs. Little work has been reported on the effect of the spatial correlation on the system performance for frequency-selective fading channels [4]. However, in practice, this idealized independent assumption does not necessarily hold due to insufficient antenna spacing and sparse scattering environment. The presence of fading channel correlation degrades the system performance. In order to combat this correlation effect, we propose a decorrelation receiving design based on the spatial decomposition proposed in [5], [6], [7], which decomposes the channel matrix of a MIMO system into a product of three matrices, transmitter configuration matrix, receiver configuration matrix, and scattering environment matrix. The transmitter configuration matrix and the receiver configuration matrix are fixed for a given antenna configuration and characterize the correlation effects on the finite separation of antennas on the overall channel matrix. The scattering environment matrix represents the parameters of the scattering region.

This spatial channel matrix decomposition [5], [6], [7] can be applied to frequency selective channels, which provides another channel equalization/estimation solution to overcome the intersymbol interference caused by multipath channels. Motivated by the correlation properties contained in the receiver configuration matrix, we propose to have a decorrelation matrix in cascade with the channel output to remove the correlations between the receivers caused by the limited antenna separations.

Another advantage of our proposed receiving technique is the function of spatial-to-mode conversion in the decorrelation matrix. Normally, with the increase of the number of antennas with constant aperture will cause increased signal correlation due to the limited antenna separation. However, this spatial-to-mode conversion virtually transforms the antenna diversity scheme to a pattern (mode) diversity scheme, where the signals obtained by different modes may be combined to yield a diversity gain [5]. The drawback of signal correlation effects between the antennas in antenna diversity scheme are simply overcome since the number of available modes is determined by the size of the receiver apertures, regardless how many antennas are packed into the apertures.

Another antenna geometry representation model - Virtual Channel Representation (VCR) model was introduced in [8], [9]. The virtual channel representation [8], [9] characterizes the channel in the spatial domain by beamforming in the direction of fixed virtual angles determined by the spatial resolution of arrays. This model allows for insights into the effects of spatial correlation and scattering. To better confirm the improved performance of our proposed design, we use this Virtual Channel Representation model as the test model to generate the channel output and then applied to our receiving system.

In this paper, we investigate the spatial aspects of the frequency-selective SIMO channel and propose a new receiving technique based on the decorrelation of the receiver signals. This receiving technique design combines the estimation/equalization technique and diversity scheme. Simulation results demonstrate the favourable system performance of this newly proposed spatial decorrelation design for frequency-selective SIMO channel.

II. System Model

Consider a frequency-selective SIMO channel with P receivers, let $\{u(k)\}$ denotes an i.i.d. scalar training sequence input to the time-varying SIMO channel, then the discrete time signal received at the pth antenna $y_p(k)$ can be written as [10]:

$$y_p(k) = \sum_{l=0}^{L} h_p(l)u(k-l) + n_p(k) \quad (1)$$

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where L is the channel-memory length, $h_p(l)$ is a discrete time representation of the channel between the transmitter and the pth receiver, and $n_p(k)$ is additive white Gaussian Noise (AWGN).

We write received signal vector

$$\boldsymbol{y}(k) = \sum_{l=0}^{L} \boldsymbol{h}(l)u(k-l) + \boldsymbol{n}(k) \tag{2}$$

where $\boldsymbol{y}(k) = [y_1(k), \ldots, y_P(k)]'$, $\boldsymbol{h}(k) = [h_1(l), \ldots, h_P(l)]'$ and $\boldsymbol{n}(k) = [n_1(k), \ldots, n_P(k)]'$.

III. Spatial Decomposition

Consider a multipath propagation scenario where there are Q multipaths originating from farfield scatters which arrive at the receiver antennas from directions $\theta_1, \theta_2, \ldots, \theta_Q$.

We assume P receivers located at polar coordinates $[(x_1, \varphi_1), (x_2, \varphi_1), \ldots, (x_P, \varphi_P)]$ from an origin and let $r_{\mathrm{R}} = \max x_1, x_2, \ldots, x_P$. Then the channel gain at the pth antenna and lth delay is given by:

$$h_p(l) = \sum_{q=1}^{Q} a_q(l) e^{j\frac{2\pi f x_P}{c}\cos(\theta_q - \varphi_P)} \tag{3}$$

where $a_q(l)$ is the gain of the signal arriving from the direction θ_q with time delay of l samples, f is the carrier frequency, c is the signal propagation speed.

We use the truncated Jocobi-Anger expansion [11], [12] to write

$$e^{2\pi f(x_P/c)\cdot\cos(\theta_q - \varphi_P)} = \sum_{m=-M}^{M} i^m J_m\big(\frac{2\pi f x_P}{c}\big) e^{-jm\theta_q} e^{jm\varphi_P} \tag{4}$$

where $M = \lceil ker/2 \rceil$, $\lceil\cdot\rceil$ is the ceiling operator, $e \approx 2.7183$ and $J_m(\cdot)$ are the Bessel functions of first kind.

We combine (3) and (4)to get

$$h_p(l) = \sum_{m=-M}^{M} \beta_m(l) J_m \frac{2\pi f x_P}{c} e^{jm\varphi_P}, \tag{5}$$

where $\beta_m(l) = \sum_{q=1}^{Q} a_q(l) i^m e^{-jm\theta_q}$. We use (5) to write,

$$\boldsymbol{h}(l) = \boldsymbol{J}_{\mathrm{R}} \boldsymbol{\beta}(l) \tag{6}$$

where $\boldsymbol{J}_{\mathrm{R}}$ is the receiver configuration matrix

$$\boldsymbol{J}_{\mathrm{R}} = \begin{pmatrix} J_{-M}(\frac{2\pi f x_1}{c}) & \ldots & J_M(\frac{2\pi f x_1}{c}) \\ \vdots & \ddots & \vdots \\ J_{-M}(\frac{2\pi f x_P}{c}) & \ldots & J_M(\frac{2\pi f x_P}{c}) \end{pmatrix}$$

and

$$\boldsymbol{\beta}(l) = [\beta_{-M}(l), \beta_{-M+1}(l), \cdots, \beta_M(l)]. \tag{7}$$

Now, we substitute (6) into (2), and get

$$\boldsymbol{y}(k) = \sum_{l=0}^{L} \boldsymbol{J}_{\mathrm{R}}(l) \boldsymbol{\beta}(l) u(k-l) + \boldsymbol{n}(k) \tag{8}$$

We have the following comments regarding the channel decomposition given by (6) :

i) Equation (6) decomposes the conventional SIMO channel vector $\boldsymbol{h}(l)$ into a product of a matrix $\boldsymbol{J}_{\mathrm{R}}$ and a vector $\boldsymbol{\beta}(l)$.

ii) The receiver configuration matrix $\boldsymbol{J}_{\mathrm{R}}$ describes the mapping of the modes of the system to the received signals. It includes antenna positions and orientations relative to the receiver origin and characterizes the correlation effect of the finite separation of antennas on the overall channel vector $\boldsymbol{h}(l)$. $\boldsymbol{J}_{\mathrm{R}}$ is known and fixed for a given receiver antenna array structure.

iii) For a random scattering environment,each $\beta_m(l)$ are random variables and time-variant with length L.

IV. Spatial Decorrelation Receiving Technique

A. Standard Receiving technique

1) Diversity Scheme: Standard receiving technique involves an antenna diversity scheme and an adaptive Normalized Least Mean Square (NLMS) channel estimator. For simplicity, the selection diversity is chosen in this paper. The strongest signal output among the receiver antennas is selected as the system output

$$z(k) = \max\{y_1(k), y_2(k), \ldots, y_P(k)\} \tag{9}$$

2) Adaptive NLMS Channel Estimator: The standard adaptive Normalised Least Mean Square (NLMS) channel estimator is employed after the system output $z(k)$. According to the diversity scheme chosen in Section (IV-A.1), we assume $\boldsymbol{h}_z(k) = [h_z(0), h_z(1), \ldots, h_z(L)]'$ is the channel vector which produces the strongest signal output $z(k)$, then the equation used to provide an estimate $\hat{\boldsymbol{h}}_z(k)$ of the unknown channel vector $\boldsymbol{h}_z(k)$ at time k is as follows [13]:

$$\hat{\boldsymbol{h}}_z(k) = \hat{\boldsymbol{h}}_z(k) + \frac{\mu}{\boldsymbol{u}(k)'\boldsymbol{u}(k) + \delta} \boldsymbol{u}(k) e(k) \tag{10}$$

$$e(k) = z(k) - \hat{\boldsymbol{h}}_z(k) \boldsymbol{u}(k) \tag{11}$$

where $\boldsymbol{u}(k) = [u(k), u(k-1), \ldots, u(k-l)]'$. Note, δ is a small positive regularization constants and μ is the step-size.

B. Spatial Decorrelation Receiving Technique

Motivated by the comments from the spatial decomposition, we design a spatial-to-modal matrix $\boldsymbol{J}_{\mathrm{R}}^{\sharp}$ in cascade with the channel output $\boldsymbol{y}(k)$ to remove the signal correlations between the receivers caused by the limited antenna separation, as shown in Figure 1. Here, we define $\boldsymbol{J}_{\mathrm{R}}^{\sharp}$ as a pseudo-inverse matrix of $\boldsymbol{J}_{\mathrm{R}}$

$$\boldsymbol{J}_{\mathrm{R}}^{\sharp} = (\boldsymbol{J}_{\mathrm{R}} \boldsymbol{J}_{\mathrm{R}}')^{-1} \boldsymbol{J}_{\mathrm{R}}' \tag{12}$$

Hence,

$$\begin{aligned} \boldsymbol{z}(k) &= \boldsymbol{J}_{\mathrm{R}}^{\sharp} \boldsymbol{y}(k) \\ &= (\boldsymbol{J}_{\mathrm{R}} \boldsymbol{J}_{\mathrm{R}}')^{-1} \boldsymbol{J}_{\mathrm{R}}' \big(\sum_{l=0}^{L} \boldsymbol{J}_{\mathrm{R}}(l) \boldsymbol{\beta}(l) u(k-l) + \boldsymbol{n}(k) \big) \\ &= \sum_{l=0}^{L} \boldsymbol{\beta}(l) u(k) + \boldsymbol{J}_{\mathrm{R}}^{\sharp} \boldsymbol{n}(k). \end{aligned} \tag{13}$$

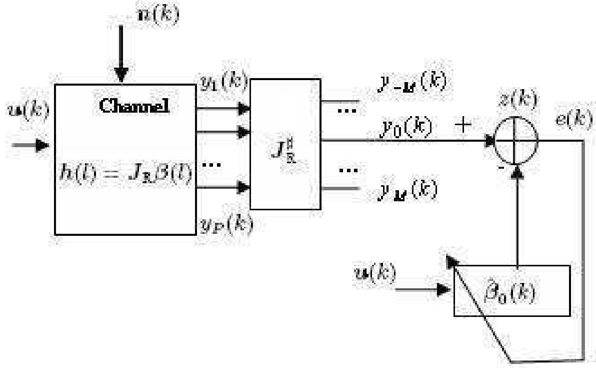

Fig. 1. System block diagram of spatially decorrelated receiver

1) Diversity Scheme: The inclusion of $\boldsymbol{J}_{\mathrm{R}}^{\sharp}$ automatically transfer the antenna diversity scheme to a pattern (mode) diversity scheme. Using the selection diversity, the system will select the strongest mode as the output, which is the zeroth order mode in this case [5], [6], [7], i.e, the Mth element of the vector $\boldsymbol{z}(k)$.

In antenna diversity scheme, by increasing the number of antennas within a constant spatial aperture will increase the signal correlations between the antennas, however, the signal correlation effect between the antennas in antenna diversity scheme are simply ignored in pattern/mode diversity scheme since the number of available modes is determined by the size of the receiver apertures, regardless how many antennas are packed into the apertures.

2) Adaptive NLMS Channel Estimator: The output $z(k)$ is then applied to the standard adaptive Normalized Least Mean Square (NLMS) channel estimator. We assume that $\boldsymbol{\beta}_0(k) = [\beta_0(0), \beta_0(1), \ldots, \beta_0(L)]'$ is the channel scattering vector which produces the strongest signal output $z(k)$, and then the equation used to provide an estimate $\hat{\boldsymbol{\beta}}_0(k)$ of the unknown channel scattering vector $\boldsymbol{\beta}_0(k)$ at time is now become:

$$\hat{\boldsymbol{\beta}}_0(k) = \hat{\boldsymbol{\beta}}_0(k) + \frac{\mu}{\boldsymbol{u}(k)'\boldsymbol{u}(k) + \delta}\boldsymbol{u}(k)e(k) \qquad (14)$$

$$e(k) = z(k) - \hat{\boldsymbol{\beta}}_0(k)\boldsymbol{u}(k) \qquad (15)$$

V. Simulations

Simulations were carried out to investigate the performance of the following two receivers: (A) Standard SIMO receiver (B) Spatially decorrelated SIMO receiver. Virtual Channel Representation channel model [8], [9] was used to generate the channel output and applied to our proposed spatially decorrelated receiver. Details of Virtual Channel Representation model can be found in Appendix

A. Simulation conditions

i) Propagation Path $L = 100$,
ii) $f = 900$MHz,

$\lvert(e(k)\rvert^2$	$\lambda/32$	$\lambda/16$	$\lambda/8$	$\lambda/4$
Receiver A	0.7758	4.083×10^{-3}	3.751×10^{-3}	3.217×10^{-3}
Receiver B	7.137×10^{-4}	5.239×10^{-4}	3.165×10^{-4}	8.336×10^{-5}

TABLE I
Average Mean Squared Error with different receiver aperture $= \lambda/32, = \lambda/16, = \lambda/8, = \lambda/4$

iii) Regularization parameter $\delta = 0.1$,
iv) Step-size $\mu = 0.01$,
v) Noise vector $\boldsymbol{n}(k)$ is zero mean Gaussian process with variance $= 0.1$,
vi) Mean squared channel estimation error $\|(e(k)\|^2$ is used to compare the system performance. All data obtained are the average of 50 similar simulations.

B. Results and Analysis

1) Simulation 1: Simulations of these two channel receivers with receiver aperture $= \lambda/4$, $\lambda/8$, $\lambda/16$,$\lambda/32$. Number of receivers $P = 5$. Results are shown in Table I, Figure 2 and Figure 3.

i) For the standard SIMO receiver (A) with receiver aperture $\lambda/32$, as the antenna spacing is insufficient, the system fails to perform the filtering operation due to the high correlations between receiver antennas.
ii) For all cases, the proposed spatially decorrelated receiver (B) show significant outperforms the standard SIMO receiver (A). This is due to the decorrelation property provided in $\boldsymbol{J}_{\mathrm{R}}^{\sharp}$.
iii) For the proposed spatially decorrelated receiver, as the number of modes is determined by the size of receiver aperture, the increase in receiver aperture increases the pattern (mode) diversity and therefore better system performance.
iv) For the standard SIMO receiver (A), the increase in receiver aperture increase the antenna separation, hence, decrease the signal correlation. However, as indicated experimentally in [14], after a certain antenna separation, further separation gives negligible improvement in signal correlation and therefore output performance.

2) Simulation 2: Simulations of these two receivers with different number of receiving antennas. The results are shown in Table II, Figure 4 and Figure 5. The receiver aperture is $= \lambda/8$.

i) For the standard SIMO receiver (A) as the number of receiving antenna within a constant aperture increases, the average mean square channel estimation error slightly drops. However, the antenna number increase coherently leads to a decrease in antenna separation which causes higher signal correlation. The system performance converges due to the increased signal correlation. This again confirms the experiment results in [14], after a certain antenna separation, further separation gives negligible

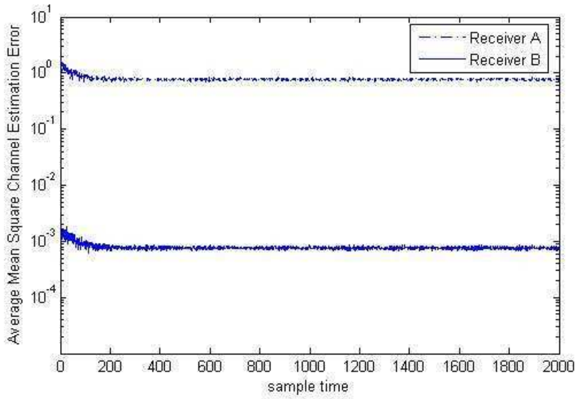

Fig. 2. Simulations for standard SIMO receiver (A) and spatially decorrelated receiver (B) with receiver aperture $= \lambda/32$.

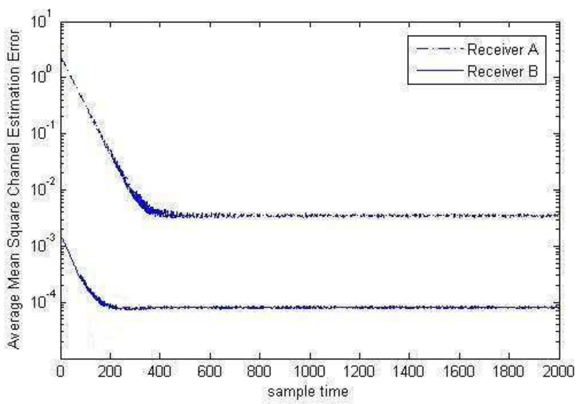

Fig. 3. Simulations for standard SIMO receiver (A) and spatially decorrelated receiver (B) with receiver aperture $= \lambda/4$.

improvement in signal correlation and therefore output performance.

ii) For the proposed spatially decorrelated receiver, as the inclusion of $\boldsymbol{J}_{\mathrm{R}}^{\sharp}$ automatically transform the antenna diversity scheme to pattern (mode) diversity scheme. Since all the modes of the antenna are independent of each other, the performance remains constant and is not affected by the increase number of antennas. This is due to the number of modes has been determined by the size of receiver aperture, regardless how many antennas are packed into the apertures.

$\|(e(k)\|^2$	Receiver A	Receiver B
$P = 3$	7.784×10^{-3}	3.264×10^{-4}
$P = 5$	3.751×10^{-3}	3.165×10^{-4}
$P = 7$	3.458×10^{-3}	3.276×10^{-4}
$P = 9$	2.921×10^{-3}	2.297×10^{-4}
$P = 11$	2.898×10^{-3}	3.306×10^{-4}

TABLE II

AVERAGE MEAN SQUARED ERROR WITH DIFFERENT NUMBER OF RECEIVER ANTENNAS

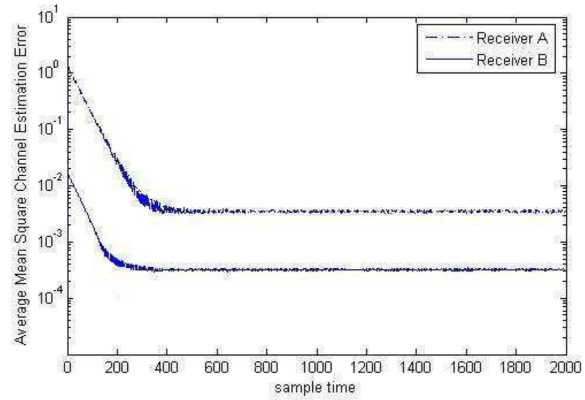

Fig. 4. Simulations for standard SIMO receiver (A) and spatially decorrelated receiver (B) with 5 receiving antennas

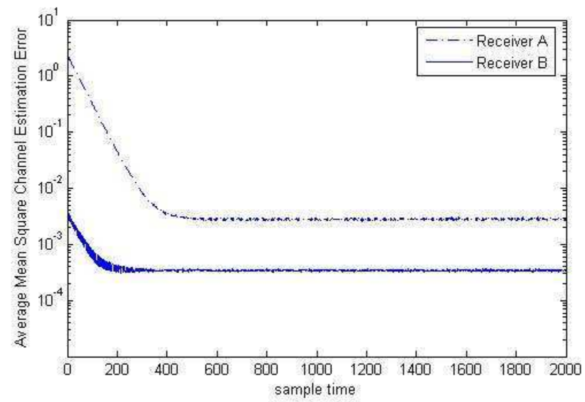

Fig. 5. Simulations for standard SIMO receiver (A) and spatially decorrelated receiver (B) with 11 receiving antennas

VI. CONCLUSIONS

In this paper, we have considered a spatially decorrelated receiving technique for the frequency selective SIMO channel. This is to combat the correlation effects between receiving antenna due to the finite antenna separation. Simulation results confirmed this favourable performance.

VII. APPENDIX

A. Virtual Channel Representation

Virtual channel representation (VCR) maps the channel matrix from antenna space to beam space. (details can be found in [8]). The time-vary channel matrix can be represented in the following form [8], [9], [15]:

$$\boldsymbol{H} = \boldsymbol{A}_R \boldsymbol{H}_v(k) \tag{16}$$

where $\boldsymbol{A}_R = [\vec{a}_R(\theta_{1R}), , \ldots, \vec{a}_R(\theta_{MR})]$. Here,

$$\vec{a}_R(\theta_{iR}) = \frac{1}{\sqrt{M}}[1, e^{-j\pi\theta_{iR}}, \ldots, e^{-j(M-1)\pi\theta_{iR})}]' \tag{17}$$

where $i \in \{1, 2, \ldots, M\}$. θ_{iR} are evenly selected within the range of $[-0.5, 0.5]$, i.e.$\theta_{iR} = 2i + 1 - M/2M$. In this case, $\boldsymbol{A}_R$ is unitary matrix. i.e. $\boldsymbol{A}_R \boldsymbol{A}_R' = I$ The virtual channel

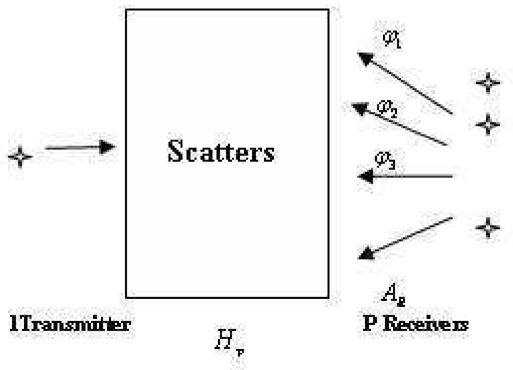

Fig. 6. Virtual Channel Representation of SIMO channel

matrix can be computed by beamforming in the direction of virtual angles.

$$\boldsymbol{H}_v, p(k) = \sum_{l=1}^{L} \beta_l(k) e^{-j\frac{2\pi f}{c} x_P \cos(\theta_{R,l} - \varphi_P)}, \qquad (18)$$

REFERENCES

[1] A.J. Paulraj and C.B. Papadias, "Space-time processing for wireless communications," *IEEE Trans. Sig. Proc.*, vol. 14, pp. 49–83, Nov. 1998.

[2] C. Komninakis, C. Fragouli, and A.H. Sayedand R.D. Wesel, "Adaptive multi-input multi-output fading channel equalization using kalman estimation," in *Proc. IEEE International conference on communications, ICC'2000*, June 2000, vol. 3, pp. 1655–1659.

[3] T. Abe and T. Matsumoto, "The finite-length multi-input multi-output mmse-dfe," *IEEE Trans. Sig. Proc.*, vol. 48, pp. 2921–2936, Oct. 2000.

[4] A.K. Sadek, W. Su, and K.J.R.Liu, "Diveristy analysis for frequency-selective mimo-ofdm systems with general spatial and temporal correlation model," *IEEE Trans. Commun.*, vol. 54, pp. 878–888, 2006.

[5] T.D. Abhayapala, T.S. Pollock, and R.A. Kennedy, "Spatial decomposition of mimo wireless channels," in *Proc. Seventh International Symposium on Signal and its applications, ISSA'2003*, 2003, pp. 309–312.

[6] T.S. Pollock, T.D. Abhayapala, and R.A. Kennedy, "Introducing space into space-time mimo capacity calculation: A new closed form upper bound," in *Proc. Int. Conf. Telecommunicatons, ICT'2003*, Papeete, Tahiti, Feb. 2003, pp. 1536–41.

[7] T.S. Pollock, T.D. Abhayapala, and R.A. Kennedy, *Spatial characterization for multiple antenna channels*, Springer, ISBN:0387228470, 2005.

[8] A.M. Sayeed, "Deconstructing multiantenna fading channels," *IEEE Trans. Sig. Proc.*, vol. 50, pp. 2563–2579, Oct. 2002.

[9] A.M. Sayeed, "A virtual representation for time-and frequency-selective correlated mimo channels," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Processing*, Apr. 2003, pp. 648–651.

[10] T. Abe and T. Matsumoto, "Space-time turbo equalizaiton in frequency-selective mimo channels," *IEEE Trans. Vehic. Technol.*, vol. 52, pp. 469–475, May 2003.

[11] H.M. Jones, R.A. Kennedy, and T.D. Abhayapala, "Spatial division multiple access in wireless communications," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Processing*, 2002.

[12] H. M. Jones, A. Saha, and T. D. Abhayapala, "The effect of finite antenna seperation on the performance of spatial diversity receivers," in *Proc. Seventh International Symposium on Signal and its applications*, July 2003, vol. 2, pp. 2837–2840.

[13] J. Homer, "Detection guided nlms estimation of sparsely parametrized channels," *IEEE Trans. Circuits and Systems–II: Analog and Digital Signal Processing*, vol. 47, pp. 1437–1442, Dec. 1998.

[14] W.C.Y. Lee, "Effects on correlation between two mobile radio base-station antennas," *IEEE Trans. Commun.*, vol. 21, no. 11, pp. 1214–1224, Nov. 1973.

[15] H. Tongand S.A. Zekavat, "A simple beamforming-simo merger in spatially correlated channel via virtual channel representation," in *IEEE Globecom 2005*, Nov. 2005, vol. 6, pp. 3544–3548.