Multirate Space-Time-Frequency Linear Block Coding

Min Zhang and Thushara D. Abhayapala
Department of Information Engineering
Australian National University
Dhammika Jayalath
School of Engineering Systems
Queensland University of Technology, Australia

David Smith
Wireless Signal Processing Program
NICTA, Australia
Chandra Athaudage
Department of Electrical Engineering,
University of Melbourne, Australia

***Abstract*—This paper presents a multirate space-time-frequency linear block coding scheme (STFBC) with full transmit diversity for a variety of transmission rates. The proposed multirate STFBC can achieve relatively smooth balance between the performance and the transmission rate for a given constellation size. Design of a space-time linear block coding (STBC) scheme is presented as a special case of the proposed multirate STFBC, which perform better than some of the existing STBCs. Moreover, optimized multirate STFBCs have also been compared with some of the existing STFBCs. Simulations results show that the design parameter has sufficient flexibility to achieve improved performance with reduced computational complexity.**

I. INTRODUCTION

The multiple input multiple output (MIMO) systems with multiple transmitters (N_t) and multiple receivers (N_r) increase the spectral efficiency of wireless communications systems. On the other hand, Orthogonal frequency division multiplexing (OFDM) combat the frequency selective fading of the channel effectively by dividing it into multiple flat fading subchannels. Both MIMO and OFDM techniques have been proposed for next generation wireless networks and has inspired the development of space-time-frequency block coding (STFBC) schemes to exploit the spatial, time and frequency diversities to improve the system performance.

There are plenty of publications on space-time block coding (STBC) in the open literature. The first ever STBC proposed in [1] can achieve full transmit and receive diversity for a MIMO system with two transmitter antennas ($N_t = 2$) and multiple receivers. The quasi-orthogonal coding for MIMO system with more than two transmitters is proposed in [2]. A rate one STBC for arbitrary number of transmitters is derived in [3]. High rate (up to N_t) full diversity STBC can improve the spectral efficiency further [4], [5].

Traditionally, a static channel for several adjacent time intervals is assumed in designing a STBC. However considering the relatively long OFDM symbol symbol duration, the STFBCs in [6]–[9] are designed assuming non-static channel between OFDM symbols. Under this assumption, the temporal and frequency domains demonstrate a similarity so that the space-frequency block coding (SFBC) can be easily extended to the STFBC. These STFBCs/SFBCs with the assumption of non-static symbol intervals require OFDM symbols with large number of subcarriers. This is a major characteristics of such STFBCs/SFBCs. The number of subcarriers has to be greater than $N_t(L+1)$ to achieve full transmit diversity, where L is the fixed channel order. The channel order gives a upper-bound for the rank of the frequency correlation matrix. Hence by employing more than "necessary" subcarriers, full spatial diversity is achieved. The channel order can be very large ($L+1 = 22$ for scenarios such as NLOS B3 channel in [10]). The order can also be time varying in real propagation scenarios. A code designed for fixed channel order would not have stable performance in such channels.

On the other hand, some STFBCs have been proposed in [11]–[13] with the assumption of static symbol intervals, or the use of adjacent subcarriers to replace adjacent symbol intervals. The major characteristics of these STFBCs and traditional STBCs is the requirement of static channel coefficients to achieve full spatial diversity. Non-static adjacent symbol intervals or subcarriers will degrade system performance at relatively low SNR range [14]. Use of more highly correlated adjacent symbol intervals or subcarriers would move this degradation into higher SNR region.

In this paper, a multirate STFBC with full transmit diversity is proposed. It provides a relatively smooth tradeoff between the performance and the transmission rate. It can also be used to design multirate STBCs. The rest of paper is organized as follows. An overview of the MIMO-OFDM channel is presented in Section II. Section III describes the proposed multirate STFBC coding scheme. The unified decoding process is presented in Section IV. Simulations results and the performance with existing codes are presented in Section V. Section VI concludes that the proposed STFBC perform better than some of the existing STBCs and adaptively change the rate depending on the quality of service (QoS) requirements.

II. MIMO-OFDM CHANNEL

Consider a MIMO-OFDM system with N_t transmitters, N_r receivers, F subcarriers and K symbol intervals ($K \geq N_t$). The frequency selective channel is assumed to be static for consecutive K OFDM symbol intervals. Each transmit and

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receive pair has $L+1$ resolvable delay paths with the same power delay profile. A block of data symbols transmitted over each transmitter passes through a F point inverse fast Fourier transform (IFFT). And a cyclic prefix (CP) is appended. The length of CP is chosen to be long enough to remove the inter symbol interference (ISI). At each receiver, the CP is removed and then a fast fourier transform (FFT) is applied. Hence, the frequency selective MIMO channel is decoupled into F parallel flat fading channels. The channel frequency response over the fth subcarrier is given by

$$\mathbf{H}(f) = \sum_{\ell=0}^{L} \mathbf{h}_\ell \times e^{-j2\pi f \Delta_f \tau_\ell} \tag{1}$$

where $f \in [1, \cdots, F]$. And τ_ℓ and $\mathbf{h}_\ell$ are the delay and complex amplitude matrix of the ℓth path between transmitters and receivers respectively. Δ_f is the subcarrier separation in the frequency domain. In this paper we assume that taps τ_ℓ are independent of each other. Moreover, the elements of $\mathbf{h}_\ell$ are assumed to be uncorrelated circularly symmetric complex Gaussian random variables with zero mean and variance σ_ℓ^2 given by the power delay profile of the channel.

Because of the relatively long OFDM symbol interval, the wireless propagation channel might not remain static during whole K consecutive symbol intervals. However the similarity between frequency and time domains reveals that consecutive subcarriers can be used to replace part of (or all) consecutive symbol intervals if the OFDM system has large number of subcarriers, eg., 1024 in [15]. Such domain switching also gives rise to high robustness to user mobility. Moreover, non-static channels degrade system performance at relatively high SNR range [14].

III. MULTIRATE STFBC CODING SCHEME

A MIMO-OFDM system is partitioned into F/P MIMO-OFDM subsystems at first in order to reduce the system complexity where P is the number of subcarriers in the MIMO-OFDM subsystem. The number of subsystems, $N_s = F/P$, is assumed to be an integer here. Each subsystem contains P well-separated subcarriers where $P \leq L+1$ so that frequency diversity and frequency correlation structure can be exploited efficiently. Therefore, full transmit diversity for a MIMO-OFDM system or subsystem is PN_t. In this paper subcarriers in the MIMO-OFDM subsystem are assumed to be independent of each other [16]. The effect of subcarrier grouping described in [11], [17] is highly related to the channel frequency correlation. Hence frequency correlation matrix $\mathbf{R}_F$, transmitter spatial correlation matrix $\mathbf{R}_{BS}$ and receiver spatial correlation matrix $\mathbf{R}_{MS}$ for the MIMO-OFDM subsystem are assumed to identity matrices.

A. Coding Process

A multirate STFBC with transmission rate R is given by:

$$\begin{aligned} s_k^q &= \mathbf{C}\mathbf{\Theta}_{Ak,R}^q + \mathbf{C}^*\mathbf{\Theta}_{Bk,R}^q; \\ \mathbf{S}_k &= \mathbf{C}\mathbf{\Theta}_{Ak,R} + \mathbf{C}^*\mathbf{\Theta}_{Bk,R}; \\ \mathbf{S} &= \mathbf{C}\mathbf{\Theta}_{A,R} + \mathbf{C}^*\mathbf{\Theta}_{B,R}; \end{aligned} \tag{2}$$

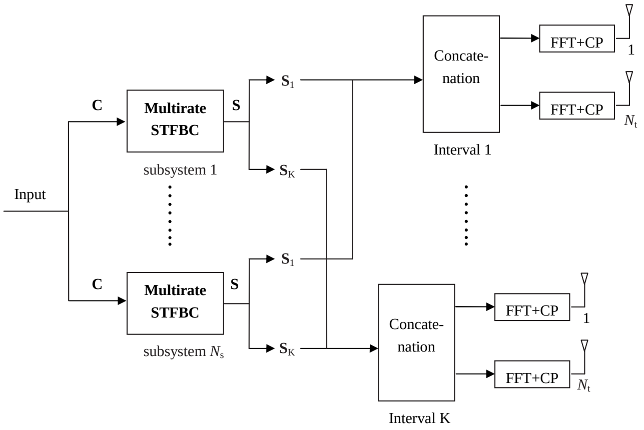

Fig. 1. Multirate STFBC processing scheme for a MIMO-OFDM system

where the codeword $\mathbf{C} = [c_1, \cdots, c_Q]$ is a $1 \times Q$ vector where $c_1, \cdots, c_Q$ are complex scalars chosen from a particularly r-PSK or r-QAM constellation $\mathcal{A}$, $Q = RPK$ and $(\cdot)^*$ is complex conjugate. Every pair of $\mathbf{\Theta}_{Ak,R}$ and $\mathbf{\Theta}_{Bk,R}$ are $Q \times N_tP$ complex coding matrices specified later. $\mathbf{\Theta}_{Ak,R}^q$ and $\mathbf{\Theta}_{Bk,R}^q$, where $q \in [1, \cdots, N_tP]$, are the qth column vectors of $\mathbf{\Theta}_{Ak,R}$ and $\mathbf{\Theta}_{Bk,R}$ respectively. $\mathbf{\Theta}_{A,R} = [\mathbf{\Theta}_{A1,R}, \cdots, \mathbf{\Theta}_{AK,R}]$ and $\mathbf{\Theta}_{B,R} = [\mathbf{\Theta}_{B1,R}, \cdots, \mathbf{\Theta}_{BK,R}]$ are $Q \times N_tPK$ matrices. Also $\mathbf{S} = [\mathbf{S}_1, \cdots, \mathbf{S}_k, \cdots, \mathbf{S}_K]$ is a $1 \times N_tPK$ matrix with $k \in [1, \cdots, K]$.

Equation (2) shows that the codeword $\mathbf{C}$ is precoded by both $\mathbf{\Theta}_{A,R}$ and $\mathbf{\Theta}_{B,R}$ matrices and dispersed from Q dimensional to N_tPK dimensional transmission data. Each $\mathbf{S}_k$ is transmitted by P subcarriers and N_t transmitters at the kth symbol interval. Moreover, the same codeword $\mathbf{C}$ is coded K times by K pairs of coding matrices $\mathbf{\Theta}_{Ak,R}$ and $\mathbf{\Theta}_{Bk,R}$. Therefore a transmission block coding scheme is specifically determined by a set of pairs of coding matrices $\mathbf{\Theta}_{Ak,R}$ and $\mathbf{\Theta}_{Bk,R}$, or a pair of $\mathbf{\Theta}_{A,R}$ and $\mathbf{\Theta}_{B,R}$. Symbol transmission rate is denoted as $R = Q/(PK)$. The value of integer Q can be chosen from 1 to N_tPK so that the symbol transmission rate R can be varied from $1/(PK)$ up to N_t. Proposed multirate STFBC processing scheme for a MIMO-OFDM system is shown in Fig. 1.

Assuming that both the real parts and the imaginary parts of $c_1, \cdots, c_Q$ have a variance of $1/2$ and are uncorrelated, we have $\mathbb{E}[c_i c_i^*] = 1$ and $\mathbb{E}[c_i^2] = 0$ where $i \in [1, \cdots, Q]$ and $\mathbb{E}[\cdot]$ is the mathematical expectation. The transmitted symbol $\mathbf{S}$ is normalized such that $\mathbb{E}[\mathbf{S}\mathbf{S}^\dagger] = N_tPK$ where $(\cdot)^\dagger$ implies the matrix adjoint of complex conjugate transpose. This leads to the following normalization equation for the proposed multirate STFBC:

$$\begin{aligned} &\text{trace}\left(\mathbf{\Theta}_{A,R}\mathbf{\Theta}_{A,R}^\dagger + \mathbf{\Theta}_{B,R}\mathbf{\Theta}_{B,R}^\dagger\right) \\ &= \sum_{k=1}^{K} \text{trace}\left(\mathbf{\Theta}_{Ak,R}\mathbf{\Theta}_{Ak,R}^\dagger + \mathbf{\Theta}_{Bk,R}\mathbf{\Theta}_{Bk,R}^\dagger\right) = N_tPK. \end{aligned} \tag{3}$$

$$\mathbf{\Lambda} = \mathbf{R}_{MS} \otimes \begin{bmatrix} \Delta\tilde{\mathbf{S}}_1 \left(\mathbf{R}_F \otimes \mathbf{R}_{BS}\right) \Delta\tilde{\mathbf{S}}_1^\dagger & \cdots & \Delta\tilde{\mathbf{S}}_1 \left(\mathbf{R}_F \otimes \mathbf{R}_{BS}\right) \Delta\tilde{\mathbf{S}}_K^\dagger \\ \vdots & \vdots & \vdots \\ \Delta\tilde{\mathbf{S}}_K \left(\mathbf{R}_F \otimes \mathbf{R}_{BS}\right) \Delta\tilde{\mathbf{S}}_1^\dagger & \cdots & \Delta\tilde{\mathbf{S}}_K \left(\mathbf{R}_F \otimes \mathbf{R}_{BS}\right) \Delta\tilde{\mathbf{S}}_K^\dagger \end{bmatrix} \tag{5}$$

B. Coding Gain

Suppose that the transmitted codeword $\mathbf{C}$ is decoded as $\tilde{\mathbf{C}}$ at the receiver with the codeword error $\Delta\mathbf{C} = \mathbf{C} - \tilde{\mathbf{C}}$, then the errors in the transmitted symbols invoked by the codeword error $\Delta\mathbf{C}$ is given by:

$$\begin{aligned} \Delta s_k^q &= \Delta\mathbf{C}\mathbf{\Theta}_{Ak,R}^q + \Delta\mathbf{C}^*\mathbf{\Theta}_{Bk,R}^q; \\ \Delta\mathbf{S}_k &= \Delta\mathbf{C}\mathbf{\Theta}_{Ak,R} + \Delta\mathbf{C}^*\mathbf{\Theta}_{Bk,R}; \\ \Delta\mathbf{S} &= \Delta\mathbf{C}\mathbf{\Theta}_{A,R} + \Delta\mathbf{C}^*\mathbf{\Theta}_{B,R}; \end{aligned} \tag{4}$$

where $\Delta\mathbf{S} = [\Delta\mathbf{S}_1, \cdots, \Delta\mathbf{S}_K]$.

The averaged pairwise error probability (PEP) between $\mathbf{C}$ and $\tilde{\mathbf{C}}$ over all channel realization can be upper bounded [18] and determined by the eigenvalues of the covariance matrix $\mathbf{\Lambda}$ given by (5) where $\Delta\tilde{\mathbf{S}}_k = (\mathbf{I}_P \otimes \mathbf{1}_{1\times N_t}) \circ (\mathbf{1}_{P\times 1} \otimes \Delta\mathbf{S}_k)$. The operators $\otimes$ and $\circ$ are defined as Kronecker product and Hadamard product respectively. Also, $\mathbf{1}_m$ and $\mathbf{1}_{m\times n}$ are defined as $m \times m$ and $m \times n$ all one matrices respectively, and $\mathbf{I}_m$ is defined as a $m \times m$ identity matrix. The covariance matrix $\mathbf{\Lambda}$ for the MIMO-OFDM subsystem with arbitrary space-time-frequency correlation structure refers to [14]. Hence with assumptions of the MIMO-OFDM subsystem, the covariance matrix $\mathbf{\Lambda}$ in (5) is simplified as

$$\mathbf{\Lambda} = \mathbf{I}_{N_r} \otimes \begin{bmatrix} \Delta\tilde{\mathbf{S}}_1\Delta\tilde{\mathbf{S}}_1^\dagger & \cdots & \Delta\tilde{\mathbf{S}}_1\Delta\tilde{\mathbf{S}}_K^\dagger \\ \vdots & \vdots & \vdots \\ \Delta\tilde{\mathbf{S}}_K\Delta\tilde{\mathbf{S}}_1^\dagger & \cdots & \Delta\tilde{\mathbf{S}}_K\Delta\tilde{\mathbf{S}}_K^\dagger \end{bmatrix} \tag{6}$$

The coding gain ζ is further defined as

$$\zeta = \arg \min_{\Delta\mathbf{C} \neq \mathbf{0}_{1\times Q}} \det(\mathbf{\Lambda})^{\frac{1}{N_r N_t P K}}. \tag{7}$$

If $P = 1$, $\Delta\tilde{\mathbf{S}}_k = \Delta\mathbf{S}_k$. Then the MIMO-OFDM subsystem is converted into a MIMO system with a single subcarrier. Thus the coding gain of the MIMO-OFDM subsystem is simplified and determined by the specific STBC coding strategy.

C. Coding Design

Previous discussion shows the primary mission of multirate STFBC design is to find the pair of coding matrices $\mathbf{\Theta}_{A,R}$ and $\mathbf{\Theta}_{B,R}$ in (2) for a given constellation and transmission rate R. Full transmit diversity N_tP is guaranteed always. Moreover, the coding gain ζ in (7) should be maximized by the coding matrices for every transmission rate.

When the MIMO-OFDM subsystem achieves the highest transmission rate N_t, $Q = N_tPK$. In this paper $\mathbf{\Theta}_{B,N_t}$ is assumed to be zero for simplicity. $\mathbf{\Theta}_{A,N_t}$ is a unitary square matrix satisfying the power constraint condition of (3) and designed as following

$$\mathbf{\Theta}_{A,N_t} = \mathbf{V}_{Q\times Q}\mathbf{D} \tag{8}$$

where $\mathbf{V}_{Q\times Q}$ is a $Q \times Q$ Vandermonde matrix given by

$$\begin{aligned} \mathbf{V}_{Q\times Q} &= [\mathbf{V}_{Q\times Q}^1, \cdots, \mathbf{V}_{Q\times Q}^i, \cdots, \mathbf{V}_{Q\times Q}^Q] \\ &= \frac{1}{\sqrt{Q}} \begin{bmatrix} 1 & \cdots & 1 & \cdots & 1 \\ \theta_1 & \cdots & \theta_i & \cdots & \theta_Q \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \theta_1^{Q-1} & \cdots & \theta_i^{Q-1} & \cdots & \theta_Q^{Q-1} \end{bmatrix} \end{aligned} \tag{9}$$

where $\mathbf{V}_{Q\times Q}^i$ is the ith column vector of square matrix $\mathbf{V}_{Q\times Q}$ and $i \in [1, \cdots, Q]$. The matrix $\mathbf{D}$ is a $Q \times Q$ diagonal matrix given by

$$\begin{aligned} \mathbf{D} = \text{Diag}(1, e^{j\phi}, \cdots, e^{j\phi \text{mod}(q+k-2,N_t)}, \cdots, \\ e^{j\phi \text{mod}(N_tP+k-2,N_t)}, \cdots, e^{j\phi \text{mod}(N_tP+K-2,N_t)}) \end{aligned} \tag{10}$$

where $q \in [1, \cdots, N_tP]$, $k \in [1, \cdots, K]$ and $\text{mod}(a, b)$ gives the remainder on division of a by b.

The coding matrix $\mathbf{\Theta}_{A,N_t}$ constitutes of the product of two complex matrices, Vandermonde matrix $\mathbf{V}_{Q\times Q}$ which guarantees to achieve full frequency diversity and diagonal matrix $\mathbf{D}$ which guarantees to achieve full spatial diversity. The design of matrix $\mathbf{V}_{Q\times Q}$ has been discussed in [3], [17] for OFDM system or MIMO system. The Vandermonde matrix specifications in [3] are used in this paper. Moreover, the coding gain ζ is optimized by the parameter ϕ of the diagonal matrix $\mathbf{D}$. For a QAM constellation and $Q = N_tPK = 2^s(s \geq 1)$, the parameters θ_i are given by $\theta_i = e^{j\frac{4i-3}{2Q}\pi}$ where $i \in [1, \cdots, Q]$ [3]. If $Q = N_tPK = 2^s \times 3^t(s \geq 1, t \geq 1)$, the parameters θ_i are given by $\theta_i = e^{j\frac{6i-5}{3Q}\pi}$. Moreover $\Delta\mathbf{C}\mathbf{V}_{Q\times Q}^i = 0$ only if $\Delta\mathbf{C} = \mathbf{0}_{1\times Q}$.

Remark 1: If $e^{j\phi}$ is an algebraic number of degree greater than $(N_t - 1)N_t$ over $\mathcal{K}$ which is the extension field containing all the entries of $\mathbf{V}_{Q\times Q}$ and the ring of complex integers $\mathbb{Z}(j)$, then full transmit diversity is guaranteed over all QAM constellations [14]. Hence it provides the freedom to design a MIMO-OFDM system with full transmit diversity.

When the bit error rate (BER) of the MIMO-OFDM subsystem is lower than the expected performance, the transmission rate R can be reduced to achieve better BER performance without decreasing in constellation size or significantly changing the coding structure. Thus $Q = RPK$ and $\mathbf{\Theta}_{A,R}$ is $RPK \times N_tPK$ matrix where $R \leq N_t$. The coding matrix $\mathbf{\Theta}_{A,R}$ for specific rate R can be obtained by simply truncating first RPK row vectors from the coding matrix $\mathbf{\Theta}_{A,N_t}$ and normalization using (3). Hence the matric $\mathbf{\Theta}_{A,R}$ is given by

$$\mathbf{\Theta}_{A,R} = \sqrt{N_t/R}\mathbf{V}_{Q\times N_tPK}\mathbf{D} \tag{11}$$

where

$$\mathbf{V}_{Q\times N_tPK} = [\mathbf{V}^1_{Q\times N_tPK}, \cdots, \mathbf{V}^i_{Q\times N_tPK}, \cdots, \mathbf{V}^{N_tPK}_{Q\times N_tPK}]$$

The vector $\mathbf{V}^i_{Q\times N_tPK}$ is the ith column vector ($i \in [1, \cdots, N_tPK]$) of matrix $\mathbf{V}_{Q\times N_tPK}$ which is truncated matrix from $\mathbf{V}_{Q\times Q}$ in (9). And $\sqrt{N_t/R}$ is the power normalization factor. The structure of matrix $\mathbf{D}$ is unchanged. However the optimal scalar ϕ_{opt} for the MIMO-OFDM subsystem can be designed for a specific rate R and symbol constellation $\mathcal{A}$. Furthermore, if $\mathbf{\Theta}_{A,N_t}$ can guarantee the full transmit diversity at rate N_t, full transmit diversity is still guaranteed at each transmission rate R without the change of ϕ. Hence if $\mathbf{\Theta}_{A,N_t}$ is specified, a series of transmission rates for the MIMO-OFDM system and corresponding coding matrices $\mathbf{\Theta}_{A,R}$ can be obtained by a direct truncation process.

Remark 2: If the coding matrix $\mathbf{\Theta}_{A,N_t}$ is capable of achieving full transmit diversity in the MIMO-OFDM system with the rate N_t, truncated coding matrix $\mathbf{\Theta}_{A,R}$ from $\mathbf{\Theta}_{A,N_t}$ can guarantee full transmit diversity for the system with a rate R. The codeword error $\Delta\mathbf{C}$ for the transmission rate R can be obtained by assigning zeros to the last $(N_t - R)PK$ symbols of $\Delta\mathbf{C}$ for the transmission rate N_t. Thus the set of $\Delta\mathbf{C}$ for the rate R actually becomes a subset of the set of $\Delta\mathbf{C}$ for the rate N_t. Therefore, for the lower transmission rate R, the subset of $\Delta\mathbf{C}$ is smaller resulting in a larger coding gain.

D. Example of the multrate STFBC

Here we show an example of multirate STFBC for the MIMO-OFDM subsystem with two transmitters, two receivers, two subcarriers and two symbol intervals. For QPSK, the coding gain ζ is maximized by computer search over ϕ varying from 0 to $\pi/2$. The step size is chosen to be $\pi/256$ so that the algebraic degree meets the condition of *Remark 1*. The optimal ϕ_{opt} are shown in Table I for a variety of transmission rates R. The scalar ϕ_{opt} is not unique in some cases. Moreover, the scalar $\phi = 119\pi/256$ gives the largest coding gain for the given MIMO-OFDM system at the highest transmission rate $R = N_t = 2$. The coding matrix $\mathbf{\Theta}_{A,R}$ obtained by direct truncation of $\mathbf{\Theta}_{A,N_t}$ also gives full transmit diversity in *Remark 2*. Hence the coding gains corresponding to $\phi = 119\pi/256$ for different rates R are also included in Table I for the comparison.

When $R = 2$, $Q = N_tKP = 8$. Then coding matrices $\mathbf{\Theta}_{Ak,2}$ for $k \in [1, 2]$ are given by

$$\begin{aligned}\mathbf{\Theta}_{A1,2} &= [\mathbf{V}^1_{8\times 8}, \phi\mathbf{V}^2_{8\times 8}, \phi\mathbf{V}^3_{8\times 8}, \mathbf{V}^4_{8\times 8}]\\ \mathbf{\Theta}_{A2,2} &= [\mathbf{V}^5_{8\times 8}, \phi\mathbf{V}^6_{8\times 8}, \phi\mathbf{V}^7_{8\times 8}, \mathbf{V}^8_{8\times 8}]\end{aligned} \tag{12}$$

where $\mathbf{V}^i_{8\times 8}$ is the ith column vector of matrix $\mathbf{V}_{8\times 8}$.

When $R = 1$, $Q = RKP = 4$. Then coding matrices $\mathbf{\Theta}_{Ak,1}$ for $k \in [1, 2]$ are given by

$$\begin{aligned}\mathbf{\Theta}_{A1,1} &= \sqrt{2}[\mathbf{V}^1_{4\times 8}, \phi\mathbf{V}^2_{4\times 8}, \phi\mathbf{V}^3_{4\times 8}, \mathbf{V}^4_{4\times 8}]\\ \mathbf{\Theta}_{A2,1} &= \sqrt{2}[\mathbf{V}^5_{4\times 8}, \phi\mathbf{V}^6_{4\times 8}, \phi\mathbf{V}^7_{4\times 8}, \mathbf{V}^8_{4\times 8}]\end{aligned} \tag{13}$$

where $\mathbf{V}^i_{4\times 8}$ is the ith column vector of matrix $\mathbf{V}_{4\times 8}$ which is constituted by truncating first four row vectors from the matrix $\mathbf{V}_{8\times 8}$. And $\sqrt{2}$ is the normalization factor after truncation.

IV. DECODING

Because of the linearity of coding in (2), channel matrices can be decomposed into their real and imaginary parts. $\mathbf{H}_D$ is constructed by stacking up P channel matrices $\mathbf{H}(f)$ in the MIMO-OFDM subsystem diagonally after subcarrier grouping and defined as

$$\mathbf{H}_D = \text{Diag}[\mathbf{H}(1), \cdots, \mathbf{H}(P)] \tag{14}$$

where $\mathbf{H}(1), \cdots, \mathbf{H}(P)$ are P channel matrices in a given MIMO-OFDM subsystem. Hence the decoding process is performed one by one subsystem. The channel equation for each symbol interval is reorganized as

$$\mathbf{Y}_k = \sqrt{\frac{\rho}{N_t}}(\mathbf{C}\mathbf{\Theta}_{Ak,R} + \mathbf{C}^*\mathbf{\Theta}_{Bk,R})\mathbf{H}_D + \mathbf{Z}_k \tag{15}$$

where $k \in [1, \cdots, K]$. $\mathbf{Y}_k$ and $\mathbf{Z}_k$ are the received signal vector and noise vector at P subcarriers during the kth symbol interval respectively. The vector $\mathbf{Z}_k$ denotes the additive white complex Gaussian (AWGN) noise with zero mean and unit variance. After some manipulations and combination, we rewrite and merge K channel complex equations into one real equation as following

$$\begin{aligned}&[\text{Re}(\mathbf{Y}_1), \text{Im}(\mathbf{Y}_1), \cdots, \text{Re}(\mathbf{Y}_K), \text{Im}(\mathbf{Y}_K)]\\ &\quad = \sqrt{\frac{\rho}{N_t}}[\text{Re}(\mathbf{C}), \text{Im}(\mathbf{C})][\mathbf{H}_{R1}, \mathbf{H}_{I1}, , \cdots, \mathbf{H}_{RK}, \mathbf{H}_{IK}]\\ &\qquad + [\text{Re}(\mathbf{Z}), \text{Im}(\mathbf{Z})]\end{aligned} \tag{16}$$

where Re and Im are real and imaginary component of a vector respectively,

$$\mathbf{H}_{Rk} = \begin{bmatrix} \text{Re}(\mathbf{\Theta}_{Ak,R}) & \text{Re}(\mathbf{\Theta}_{Bk,R}) & -\text{Im}(\mathbf{\Theta}_{Ak,R}) & -\text{Im}(\mathbf{\Theta}_{Bk,R}) \\ -\text{Im}(\mathbf{\Theta}_{Ak,R}) & \text{Im}(\mathbf{\Theta}_{Bk,R}) & -\text{Re}(\mathbf{\Theta}_{Ak,R}) & \text{Re}(\mathbf{\Theta}_{Bk,R}) \end{bmatrix} \times \begin{bmatrix} \text{Re}(\mathbf{H}_D) \\ \text{Re}(\mathbf{H}_D) \\ \text{Im}(\mathbf{H}_D) \\ \text{Im}(\mathbf{H}_D) \end{bmatrix}$$

and

$$\mathbf{H}_{Ik} = \begin{bmatrix} \text{Im}(\mathbf{\Theta}_{Ak,R}) & \text{Im}(\mathbf{\Theta}_{Bk,R}) & \text{Re}(\mathbf{\Theta}_{Ak,R}) & \text{Re}(\mathbf{\Theta}_{Bk,R}) \\ \text{Re}(\mathbf{\Theta}_{Ak,R}) & -\text{Re}(\mathbf{\Theta}_{Bk,R}) & -\text{Im}(\mathbf{\Theta}_{Ak,R}) & \text{Im}(\mathbf{\Theta}_{Bk,R}) \end{bmatrix} \times \begin{bmatrix} \text{Re}(\mathbf{H}_D) \\ \text{Re}(\mathbf{H}_D) \\ \text{Im}(\mathbf{H}_D) \\ \text{Im}(\mathbf{H}_D) \end{bmatrix}.$$

R	ϕ_{opt}	$\zeta(\phi = \phi_{opt})$	$\zeta\,(\phi = 119\pi/256)$
1/4	$\pi/2$	3.1623	3.1623
2/4	$\pi/2$	2	1.9878
3/4	$87\pi/256$	1.1648	0.8868
1	$\pi/2$	0.6436	0.5760
5/4	$121\pi/256$	0.2757	0.2103
6/4	$127\pi/256$	0.2211	0.1752
7/4	$126\pi/256$	0.1290	0.1024
2	$119\pi/256$	0.0896	0.0896

TABLE I
TRANSMISSION RATE R VS CODING GAIN ζ FOR $\phi = \phi_{opt}$, AND $\phi = 119\pi/256$ IN THE MIMO-OFDM SYSTEM WITH $N_t = N_r = P = K = 2$ AND QPSK.

The equation (16) gives a unified decoding equation with $2Q$ dimensions and real parameters for a variety of transmission rates R. Moreover, to achieve the full transmit and receive diversity performance, the maximum-likelihood detection (MLD) should be used for the multirate STFBC. Considering high transmission rate R and large value of $Q = RPK$, sphere decoding [19] is adopted in this paper to achieve near MLD performance.

V. SIMULATIONS AND COMPARISONS

In this section, BER performance of a MIMO-OFDM system with two transmitters, two receivers and 512 subcarriers is investigated through computer simulations. The random channel is assumed to be a multipath channel with a uniform delay profile composed of $L+1$ independently identically distributed complex Gaussian paths with zero mean and equal variance of $1/(L+1)$. We choose $L = 3$ and $P = 2$. Subcarriers for each MIMO-OFDM subsystem are well-separated. The second order characteristics of such MIMO-OFDM subsystem is presented in [16]. All STFBCs/STBCs are reformatted to a real equation with (16) during decoding and the sphere decoding is used at the receiver [19].

If $P = 1$, the MIMO-OFDM subsystem is actually a narrowband MIMO system. Hence a STBC is a special case of STFBC. The coding matrix (8) for the STFBC can be used to design a STBC and then compared with other STBCs in [4], [5], which also can be unified into the same structure of (2). These STBCs have same transmission rate of two. The scalar ϕ in the coding matrix (8) for the STBC is optimized for the 2×2 MIMO system. It gives $\phi_{opt} = 73\pi/256$ if QPSK constellation is employed and correspondingly coding gain $\zeta = 0.8203$. For comparison, the golden code in [5] and high rate STBC in [4] give the coding gain $\zeta = 0.9457$ and $\zeta = 0.6883$ respectively. The coding gain defined in this paper is slightly different to [4], [5]. Simulations for these STBCs in the MIMO-OFDM system are shown in Fig. 2 where frequency diversity is not achieved since $P = 1$. It is shown that the STBC with the coding matrix (8) has performance very close to the golden code in [5] (about 0.1dB difference), but better performance than high rate STBC in [4] (about 0.8dB difference). Simulations confirm that the coding matrix (8) can achieve relatively large coding gain and good performance compared with some existing STBCs.

The multirate STFBC following the coding matrix (11) is investigated in Fig. 3 for the MIMO-OFDM system with $N_t = N_r = P = K = 2$ and QPSK constellation. The scalar $\phi = \phi_{opt}$ is specified in Table I for a variety of transmission rate R. It is shown that with the decrease of transmission rate, the coding gain is increased and consequently the BER performance is better. The SNR gain is roughly 1.5dB for each decrease of transmission rate. Hence the coding matrix (11) shows a flexible structure so that targeted BER performance can be achieved by smoothly reducing the transmission rate. Moreover, in Fig. 3 the multirate STFBC is also compared with the rate one STFBC in [11] with QPSK and 16QAM constellation. The STFBC in [11] with 16QAM is compared with the multirate STFBC with $R = 2$ since both STFBCs have same spectral efficiency of 4 bit/s/Hz. It is shown that the multirate STFBC with $R = 2$ has about 2.5dB gain. On the other hand, the STFBC in [11] with QPSK is compared with the multirate STFBC with $R = 1$ since both STFBCs have same spectral efficency of 2 bit/s/Hz. It is shown that the multirate STFBC with $R = 1$ has about 0.3dB loss of BER performance because the efficient rate one Alamouti code is embedded in the STFBC [11]. However, the change of constellation size from QPSK to 16QAM for the STFBC [11] leads to a significant degradation of system performance. Hence, the multirate STFBC can achieve a smoother balance between the transmission rate and BER performance.

The effect of the scalar ϕ on the coding gain ζ is shown in Fig. 4. The coding gain in (7) for the MIMO-OFDM subsystem is calculated for QPSK constellation. The multirate STFBC with $R = 1$ gives larger and smoother coding gain than the multirate STFBC with $R = 2$ because of lower transmission rate. Moreover, simulations show that the full transmit diversity of the MIMO-OFDM system is achieved for every scalar ϕ. Hence with the increase of algebraic degree of ϕ, the full diversity is easily achieved in the MIMO-OFDM system. Both lines in Fig. 4 show repeated deep troughs in the coding gain. The oscillations between deep troughs are relatively small. Hence in general the BER performance is not significantly sensitive to the scalar ϕ for a ranges of values between these troughs. Therefore some particular and easily calculated values, eg. $\phi = \pi/2$ and $e^{j\phi} = j$, can be chosen without seriously sacrificing the system performance.

VI. CONCLUSION

The design of a multirate STFBC scheme with full transmit diversity is presented in this paper. A variety of rate adaptive coding matrices are obtained by a simple truncation of the coding matrix, $\mathbf{\Theta}_{A,N_t}$, or by parameter optimization for coding matrices for a given transmission rate and constellation. The structure of the STFBC is also used to design STBCs, which show a close or better performance than some of the existing STBCs. The proposed multirate STFBC can achieve relatively

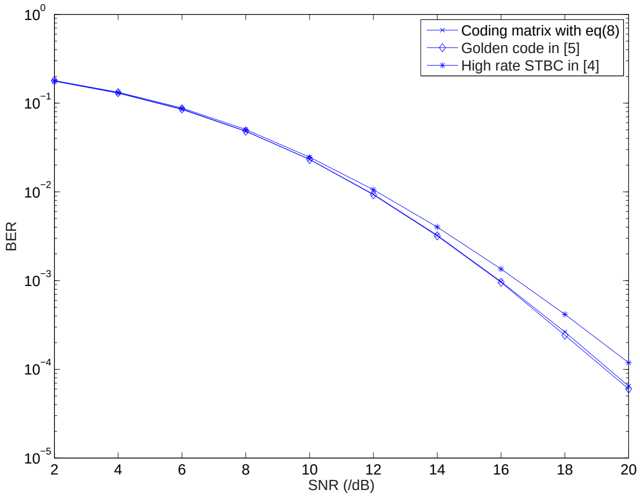

Fig. 2. Comparisons of BER performance of the coding matrix (8), the golden code [5] and high rate STBC [4] for a MIMO-OFDM system where $N_t = N_r = K = 2$ and $P = 1$ and QPSK

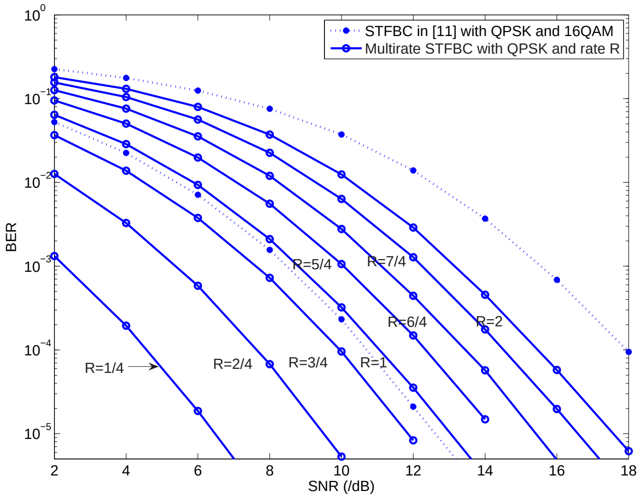

Fig. 3. Multirate STFBC simulations with $\phi = \phi_{opt}$ for the MIMO-OFDM system where $N_t = N_r = K = P = 2$

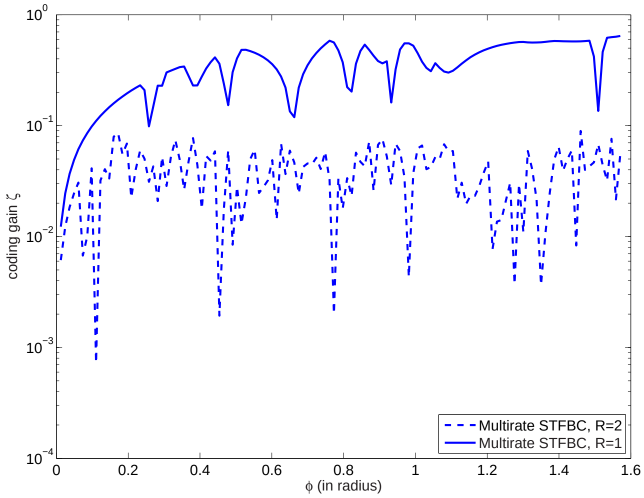

Fig. 4. Coding gain ζ vs the scalar ϕ with QPSK for the MIMO-OFDM system where $N_t = N_r = K = P = 2$

smooth balance between the transmission rate and the performance without the need to change constellation size. It has a relatively smooth coding gain for small variations in the design parameter giving us the flexibility to select certain special values close to the optimum design parameter to reduce the computation complexity without sacrificing the performance significantly.

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